CANKAYA UNIVERSITY
FACULTY OF ENGINEERING AND ARCHITECTURE
MECHANICAL ENGINEERING DEPARTMENT
ME 212 THERMODYNAMICS II

CAHPTER 9

EXAMPLES SOLUTION

1) An air-standard Otto cycle has a compression ratio of 8.5. At the beginning of compression, $p_1 = 100$ kPa and $T_1 = 300$ K. The heat addition per unit mass of air is 1400 kJ/kg. Determine

a) The net work, in kJ per kg of air.
b) The thermal efficiency of the cycle.
c) The mean effective pressure, in kPa
d) The maximum temperature in the cycle, in K

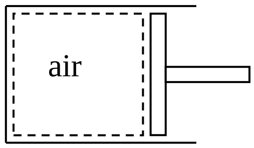
To investigate the effects of varying compression ratio, plot each of the quantities calculated in parts (a) through (d) for compression ratios ranging from 1 to 12.

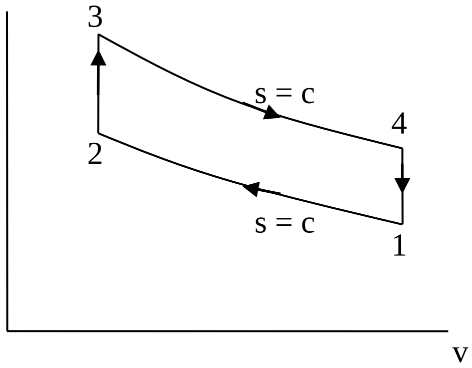
Solution:

Known: An air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition per unit mass of air is given.

Find: Determine (a) the net work per unit mass of air, (b) the thermal efficiency, (c) the mean effective pressure, and (d) the maximum cycle temperature. Plot each of these quantities versus compression ratio.

Schematic & Given Data:





$V_1/V_2 = 8.5$

$\frac{Q_{23}}{m} = 1400 \text{ kJ/kg}$

$p_1 = 100$ kPa

$T_1 = 300$ K

Assumptions:

1. The air in the piston-cylinder assembly is the closed system.
2. The compression and expansion processes are adiabatic.
3. All processes are internally reversible
4. The air is modeled as an ideal gas
5. Kinetic and potential energy effects are negligible

Analysis: Begin by fixing each principle state of the cycle

State 1:

$$p_1 = 100 \text{ kPa}, \ T_1 = 300\text{K} \Rightarrow u_1 = 214.07 \text{ kJ/kg}, \ v_{r1} = 621.2$$

State 2:

For isentropic compression

$$v_{r2} = v_{r1}\frac{V_2}{V_1} = \frac{621.2}{8.5} = 73.082$$

$$\text{Thus}, \ T_2 = 688.2 \text{ K}, \ u_2 = 503.06 \text{ kJ/kg}$$

State 3:

The specific internal energy u_3 is found by using the energy balance for process 2-3

$$m(u_3 - u_2) = Q_{23} - \underbrace{W_{23}}_{0}$$

$$u_3 = \frac{Q_{23}}{m} + u_2 = 1400\frac{kJ}{kg} + 503.06\frac{kJ}{kg} = 1903.06\frac{kJ}{kg}$$

Thus, $T_3 = 2231.3\text{ K},\ v_{r3} = 1.9192$

State 4:

For the isentropic expansion

$$v_{r4} = v_{r3}\frac{V_4}{V_3} = v_{r3}\frac{V_1}{V_2} = (1.9192)(8.5) = 16.3132$$

Finally, $T_4 = 1154.3\text{ K},\ u_4 = 892.91\ \text{kJ/kg}$

(a) To find the net work, note that $W_{cycle} = Q_{cycle}$, so

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = \frac{Q_{23}}{m} - (u_4 - u_1)$$
$$= 1400 - (892.91 - 214.07) = 721.16\ \frac{kJ}{kg}$$

(b) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{721.12\ (kJ/kg)}{1400\ (kJ/kg)} = 0.515\ (51.5\%)$$

(c) The displacement volume is $V_1 - V_2 = m(v_1 - v_2)$, so the mean effective pressure is given by

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1 - v_2} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)}$$

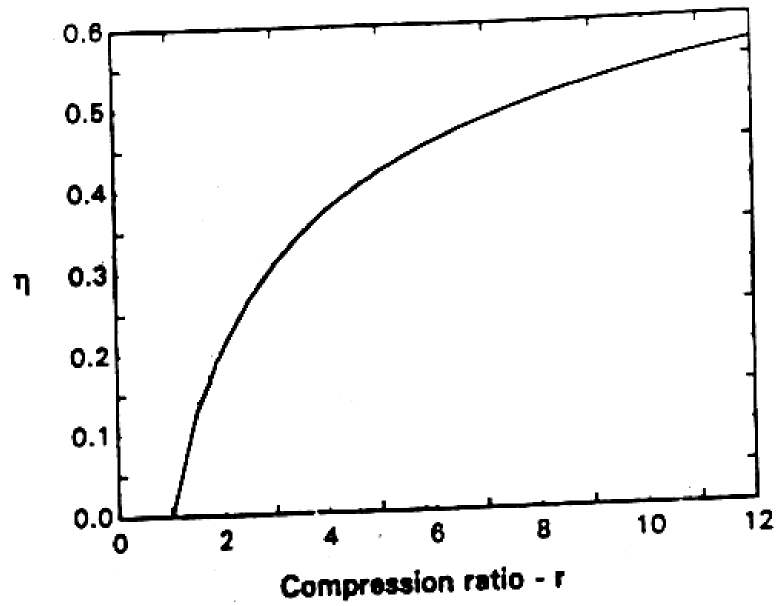
Evaluating v_1

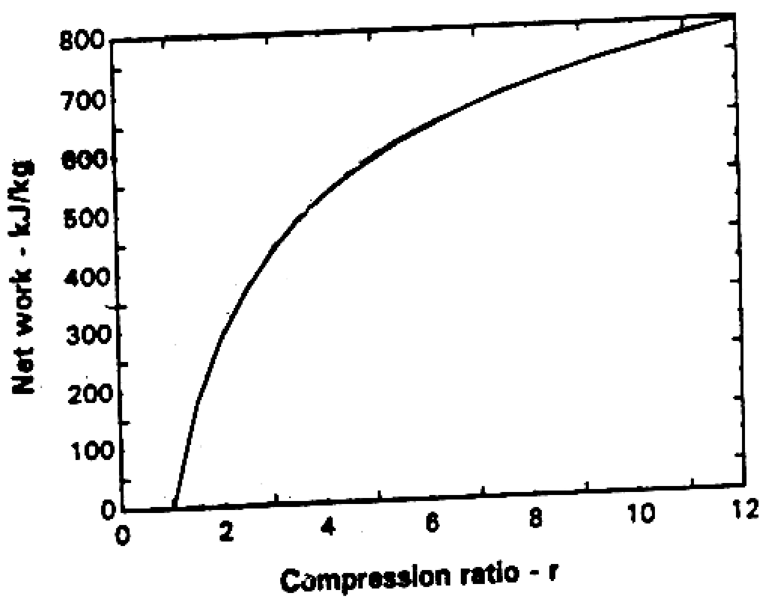
$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{8.314}{28.97}\right)\frac{kJ}{kgK}(300\,K)}{(100\ kPa)}\left(\frac{1\ kPa}{10^3\ N/m^2}\right)\left(\frac{10^3\ N.m}{1\ kJ}\right) = 0.861\ m^3/kg$$

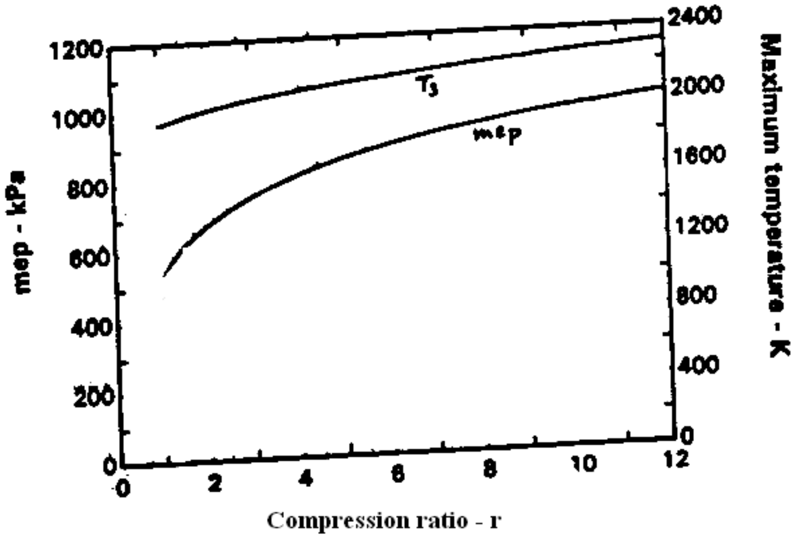
Thus

$$mep = \frac{(721.12\ kJ/kg)}{\left(0.861\ \frac{m^3}{kg}\right)\left(1 - \frac{1}{8.5}\right)}\left(\frac{10^3\ N.m}{1\ kJ}\right)\left(\frac{1\ kPa}{10^3\ N/m^2}\right)$$
$$= 949.2\ kPa$$

Plotting for compression ranging from 1 to 12,







mep - kPa
Maximum temperature - K
T_3
mep
0
200
400
600
800
1000
1200
0
400
800
1200
1600
2000
2400
0
2
4
6
8
10
12
Compression ratio - r

3) The pressure and temperature at the beginning of compression of an air-standard Diesel cycle are 95 kPa and 290 K, respectively. At the end of the heat addition, the pressure is 6.5 MPa and the temperature is 2000 K.
Determine

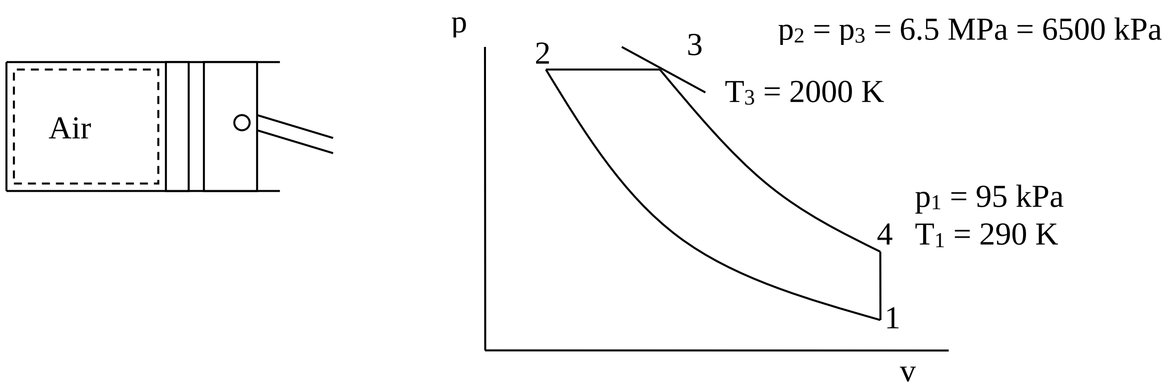
a) The compression ratio.
b) The cut off ratio.
c) The thermal efficiency of the cycle.
d) The mean effective pressure, in kPa.

Solution:

Known: An air-standard Diesel cycle has a specified state at the beginning of compression and a known pressure and temperature at the end of heat addition.

Find: Determine (a) the compression ratio, (b) the cut off ratio, (c) the thermal efficiency, and (d) the mean effective pressure

Schematic and Given Data:



Assumptions:

1) The air in the piston-cylinder assembly is the closed system.
2) The compression and expansion processes are adiabatic.
3) All processes are internally reversible.
4) The air is modeled as an ideal gas.
5) Kinetic and potential energy effects are negligible.

Analysis: Begin by fixing each principal state in the cycle.

State 1:

$$T_1 = 290\text{ K},\ p_1 = 95\,\text{kPa} \Rightarrow u_1 = 206.91\text{ kJ/kg},\ v_{r1} = 676.1,\ p_{r1} = 1.2311$$

State 2: For the isentropic compression

$$p_{r2} = p_{r1}\left(\frac{p_2}{p_1}\right) = (1.2311)\left(\frac{6500}{95}\right) = 84.233 \Rightarrow v_{r2} = 31.59,\ T_2 = 926\ K,\ h_2 = 962.19\ kJ/kg$$

State 3:

$$T_3 = 2000\ K,\ \ P_3 = 6500\,kPa \Rightarrow h_3 = 2252.1\ kJ/kg,\ v_{r3} = 2.776$$

State 4:

For the isentropic expansion,

$$\frac{V_4}{V_3} = \frac{V_1}{V_2}\frac{V_2}{V_3} = \frac{V_1}{V_2}\frac{T_2}{T_3} = \frac{v_{r1}}{v_{r2}}\frac{T_2}{T_3} = \frac{676.1}{31.59}\frac{926}{2000} = 9.9$$

$$v_{r4} = \frac{V_4}{V_3}v_{r3} = 27.48 \Rightarrow T_4 = 971\ K,\ \ u_4 = 734.36\ kJ/kg$$

(a) The compression ratio is

$$r = \frac{V_1}{V_2} = \frac{v_{r1}}{v_{r2}} = \frac{676.1}{31.59} = 21.4$$

b) The cut off ratio

$$r_c = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2000}{926} = 2.16$$

c) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{(h_3 - h_2) - (u_4 - u_1)}{h_3 - h_2} = \frac{(2252.1 - 962.19) - (734.36 - 206.91)}{(2252.1 - 962.19)}$$

$$= \frac{762.46}{1289.91} = 0.591\ (59.1\%)$$

d) The mean effective pressure is given as

$$\text{mep} = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)}$$

evaluating v_1

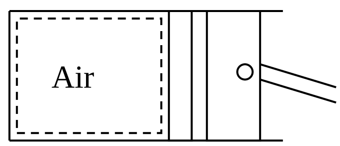
$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg.K}}\right)(290\,\text{K})}{(95\text{ kPa})}\left(\frac{1\text{ kPa}}{10^3\ \text{N/m}^2}\right)\left(\frac{10^3\text{ Nm}}{1\text{ kJ}}\right) = 0.8761\text{ m}^3/\text{kg}$$

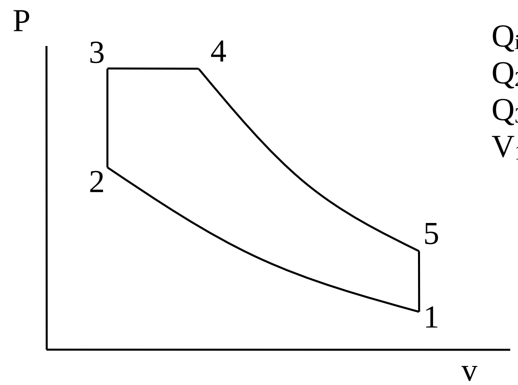
Thus

$$\text{mep} = \frac{(762.46\text{ kJ/kg})}{\left(0.8761\,\frac{\text{m}^3}{\text{kg}}\right)\left(1 - \frac{1}{21.4}\right)}\left(\frac{10^3\ \text{Nm}}{1\text{ kJ}}\right)\left(\frac{1\text{ kPa}}{10^3\ \text{N/m}^2}\right)$$

$$= 913\text{ kPa}$$

6) An air-standard dual cycle has a compression ratio of 9. At the beginning of compression, $p_1 = 100$ kPa and $T_1 = 300\,\text{K}$. The heat addition per unit mass of air is 1400 kJ/kg, with two thirds added at constant volume and the rest at constant pressure.





$Q_{in}/m = 1400$ kJ/kg
$Q_{23}/m = 933.3$ kJ/kg
$Q_{34}/m = 466.7$ kJ/kg
$V_1/V_2 = 9$

$p_1 = 100$ kPa
$T_1 = 300$ K

Determine

a) The temperatures at the end of each heat addition process, in K.
b) The net work of the cycle per unit mass of air, in kJ/kg
c) The thermal efficiency.
d) The mean effective pressure, in kPa.

Solution:

Known: An air-standard dual cycle has a known compression ratio and a specified state at the beginning of compression. The heat additions at constant volume and constant pressure are also given.

Find: Determine (a) the temperatures at the end of each heat addition process, (b) the net work per unit mass, (c) the thermal efficiency, and (d) the mean effective pressure.

Schematic and Given Data:

Assumptions:

1) The air in the piston-cylinder assembly is the closed system.
2) The compression and expansion processes are adiabatic.
3) All processes are internally reversible.
4) The air is modeled as an ideal gas.
5) Kinetic and potential energy effects are negligible.

Analysis: Begin by fixing principal state of the cycle

State 1:

$$T_1 = 300\text{ K} \Rightarrow u_1 = 214.07\text{ kJ/kg},\ \ v_{r1} = 621.2$$

State 2:

For the isentropic compression, $v_{r2} = (V_2 / V_1)v_{r1} = 69.022$
Thus, $T_2 = 702.7$ K and $u_2 = 514.4$ kJ/kg

State 3: For the heat addition process from 2 to 3:

$$m(u_3 - u_2) = Q_{23} - \underbrace{W_{23}}_{0}$$

or

$$u_3 = Q_{23}/m + u_2 = 933.3 + 514.4 = 1447.7\text{ kJ/kg}$$

(a) Thus, $T_3 = 1758.5$ K and $h_3 = 1952$ kJ/kg

State 4:
For the heat addition process from 3 to 4:

$$Q_{34}/m = h_4 - h_3$$

or

$$h_4 = h_3 + Q_{34}/m = 1952 + 466.7 = 2418.7\text{ kJ/kg}$$

Thus, $T_4 = 2132.8$ K and $v_{r4} = 2.237$

State 5: For the isentropic expansion

$$v_{r5} = (V_5 / V_4)v_{r4} = \left(\frac{V_1}{V_2}\frac{V_3}{V_4}\right)v_{r4} = \left(\frac{V_1}{V_2}\frac{T_3}{T_4}\right)v_{r4} = 16.5997$$

Thus, $T_5 = 1147.8$ K and $u_5 = 887.24$ kJ/kg

(b) For the cycle, $W_{cycle} = Q_{cycle}$. Thus

$$W_{cycle}/m = \left(Q_{23}/m + Q_{34}/m\right) - Q_{51}/m = Q_{in}/m - \left(u_5 - u_1\right)$$
$$= 1400 - (887.24 - 214.07) = 726.83\text{ kJ/kg}$$

c) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{in}/m} = \frac{726.83}{1400} = 0.519\ \ (51.9\%)$$

d) The mean effective pressure is given by

$$\text{mep} = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kgK}}\right)(300\ \text{K})}{(100\ \text{kPa})}\left(\frac{1\ \text{kPa}}{10^3\ \text{N/m}^2}\right)\left(\frac{10^3\ \text{Nm}}{1\ \text{kJ}}\right) = 0.86096\ \text{m}^3/\text{kg}$$

Thus

$$\text{mep} = \frac{(726.83\ \text{kJ/kg})}{(0.86096\ \text{m}^3/\text{kg})(1 - 1/9)}\left(\frac{10^3\ \text{Nm}}{1\ \text{kJ}}\right)\left(\frac{1\ \text{kPa}}{10^3\ \text{N/m}^2}\right)$$

$$= 949.7\ \text{kPa}$$

6) Air enters the compressor of an ideal air-standard Brayton cycle at 100 kPa, 300 K, with a volumetric flow rate of 5 m^3/s. The compressor pressure ratio is 10. For turbine inlet temperatures ranging from 1000 to 1600 K, plot

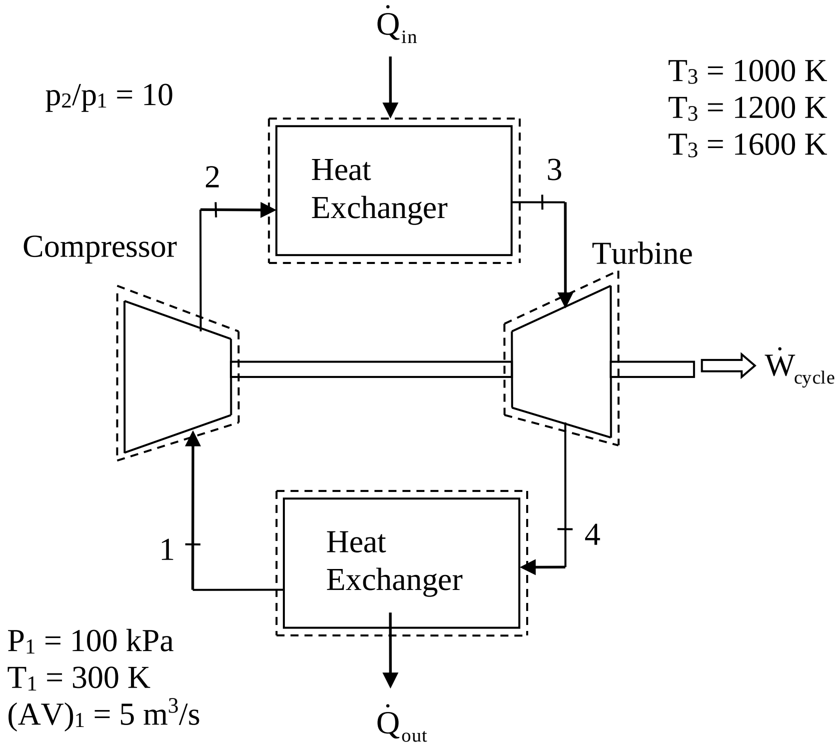
a) The thermal efficiency of the cycle.
b) The back work ratio.
c) The net power developed, in kW.

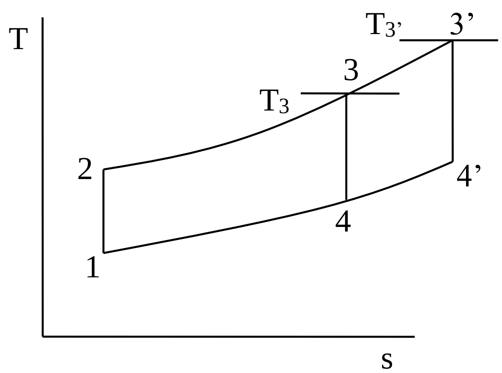
Solution:

Known: Air enters the compressor of an ideal Brayton cycle with known conditions. The compressor pressure ratio is also known.

Find: Plot for various turbine inlet temperatures (a) the thermal efficiency, (b) the back work ratio, (c) the net power developed.

Schematic and Given Data:





Assumptions:

1) Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2) The turbine and compressor processes are isentropic
3) There are no pressure drops for flow through the heat exchangers.
4) Kinetic and potential energy effects are negligible.
5) The working fluid is air modeled as an ideal gas.

Analysis: Sample calculations are given below for several turbine inlet temperatures. First, fix each of the principal states.

State 1: $T_1 = 300\ \text{K} \Rightarrow h_1 = 300.19\ \text{kJ/kg},\ pr_1 = 1.3860$

State 2: For the isentropic compression, $p_{r2} = (p_2/p_1)pr_1 = 13.860$

Thus, $T_2 = 574.1\ \text{K}$ and $h_2 = 579.86\ \text{kJ/kg}$

State 3:

$$T_3 = \begin{cases} 1000\ \text{K} \Rightarrow h_3 = 1046.04\ \text{kJ/kg},\ pr_3 = 114.0 \\ 1200\ \text{K} \Rightarrow h_3 = 1277.79\ \text{kJ/kg},\ pr_3 = 238.0 \\ 1600\ \text{K} \Rightarrow h_3 = 1757.57\ \text{kJ/kg},\ pr_3 = 791.2 \end{cases}$$

State 4: For the isentropic expansion, $pr_4 = (p_4/p_3)pr_3 \Rightarrow T_4, h_4$

Thus

$$pr_4 = \begin{cases} 11.4, & T_4 = 543.9\ \text{K}, & h_4 = 548.45\ \text{kJ/kg} & (1000\,\text{K}) \\ 23.8, & T_4 = 665\ \text{K}, & h_4 = 675.84\ \text{kJ/kg} & (1200\,\text{K}) \\ 79.12, & T_4 = 911.3\ \text{K}, & h_4 = 945.65\ \text{kJ/kg} & (1600\,\text{K}) \end{cases}$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{h_4 - h_1}{h_3 - h_2}$$

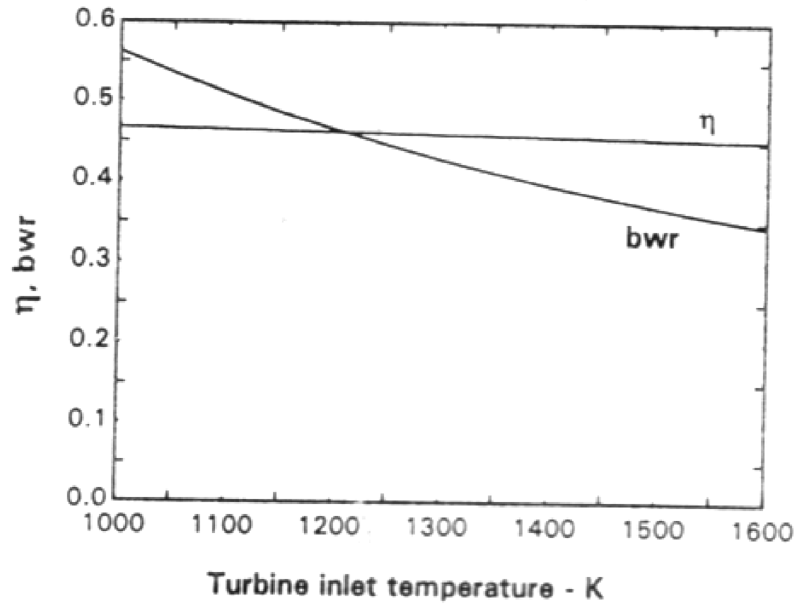
(b) The back work ratio is

$$\text{bwr} = \frac{h_2 - h_1}{h_3 - h_4} = \frac{279.67}{h_3 - h_4}$$

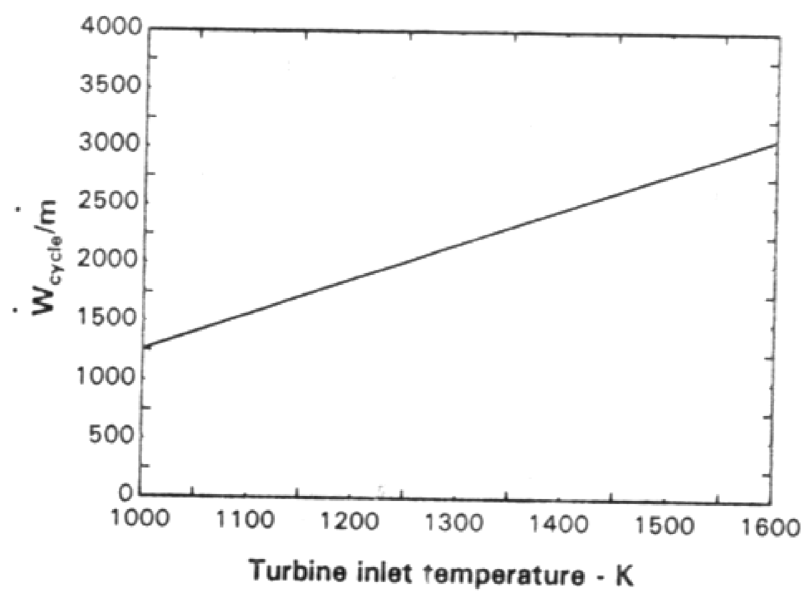
c) The power is

$$\dot{W}_{net} = \dot{m}\left[(h_3 - h_4) - (h_2 - h_1)\right] = \frac{(AV)_1 p_1}{RT_1}\left[(h_3 - h_4) - 279.67\right]$$

The results are summarized in the plots, which give each of these quantities versus turbine inlet temperatures ranging from 1000 to 1600K.



0.6
0.5
0.4
0.3
0.2
0.1
0.0
η, bwr
η
bwr
1000
1100
1200
1300
1400
1500
1600
Turbine inlet temperature - K



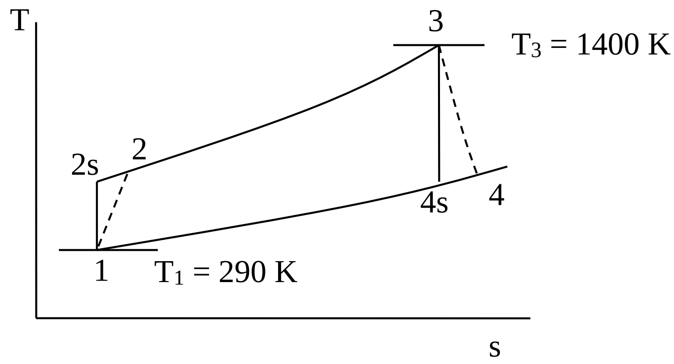
4000
3500
3000
2500
2000
1500
1000
500
0
$\dot{W}_{cycle}/\dot{m}$
1000
1100
1200
1300
1400
1500
1600
Turbine inlet temperature - K

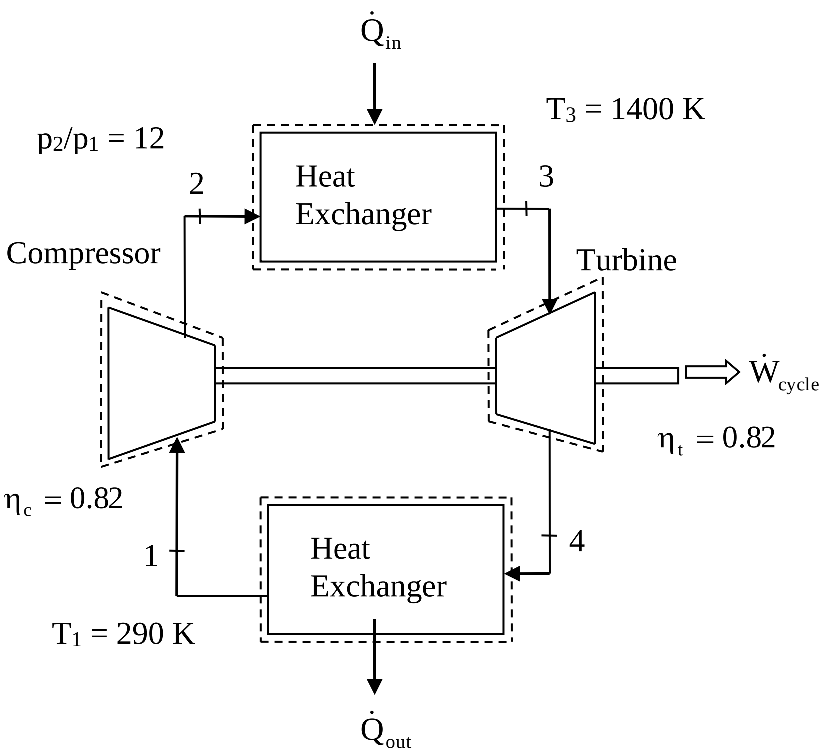
7) The compressor and turbine of a simple gas turbine each have isentropic efficiencies of 82%. The compressor pressure ratio is 12. The minimum and maximum temperatures are 290 K and 1400 K, respectively. On the basis of an air-standard analysis, compare the values of

a) The net work per unit mass of air flowing, in kJ/kg.
b) The heat rejected per unit mass of air flowing, in kJ/kg, and
c) The thermal efficiency to the same quantities evaluated for an ideal cycle.

Solution:
Known: An air-standard gas turbine cycle has a known compressor pressure ratio and specified minimum and maximum temperatures. The compressor and turbine each have isentropic efficiencies of 82%.
Find: Determine (a) the net work per unit mass of air flow, (b) the heat rejected per unit mass of air flow, and (c) the thermal efficiency and compare them to the same quantities evaluated for an ideal cycle.

Schematic and Given Data:





Assumptions:

1) Each component is analyzed as a control volume at steady state.
2) The compressor and turbine are adiabatic.
3) There are no pressure drops for flow through the heat exchangers.
4) Kinetic and potential energy effects are negligible.
5) The working fluid is air modeled as an ideal gas.

Analysis: First, fix each of the principal states for each cycle using data from Tables.

State 1:

$$T_1 = 290 \text{ K} \Rightarrow h_1 = 290.16 \text{ kJ/kg}, \quad p_{r1} = 1.2311$$

State 2: First, determine h_{2s}. For isentropic compression

$$p_{r2s} = (p_2/p_1)p_{r1} = 14.7732 \Rightarrow h_{2s} = 590.47 \text{ kJ/kg}$$

Using the compressor efficiency; $\eta_c = (h_{2s} - h_1)/(h_2 - h_1)$;

$$h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 656.39 \text{ kJ/kg}$$

State 3;

$$T_3 = 1400 \text{ K} \Rightarrow h_3 = 1515.42 \text{ kJ/kg}, \quad p_{r3} = 450.5$$

State 4: First, determine h_{4s}. For isentropic expansion

$$p_{r4s} = (p_4/p_3)P_{r3} = 37.542 \Rightarrow h_{4s} = 768.38 \text{ kJ/kg}$$

Using the turbine efficiency; $\eta_t = (h_3 - h_4)/(h_3 - h_{4s})$;

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) \Rightarrow h_4 = 902.85 \text{ kJ/kg}$$

(a) The net work per unit mass of air flowing is

$$\frac{\dot{W}_{cycle}}{\dot{m}} = (h_3 - h_4) - (h_2 - h_1) = (1515.4 - 902.85)\frac{\text{kJ}}{\text{kg}} - (656.39 - 290.16)\frac{\text{kJ}}{\text{kg}}$$
$$= 246.3 \text{ kJ/kg}$$

For the ideal cycle

$$\left(\frac{\dot{W}_{cycle}}{\dot{m}}\right)_{ideal} = (h_3 - h_{4s}) - (h_{2s} - h_1) = 446.7 \text{ kJ/kg}$$

Thus, irreversibilities reduce the net work by nearly one half

(b) The heat rejected per unit mass of air flowing is

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_4 - h_1 = 902.85\frac{\text{kJ}}{\text{kg}} - 290.16\frac{\text{kJ}}{\text{kg}} = 612.7 \text{ kJ/kg}$$

For the ideal cycle

$$\left(\frac{\dot{Q}_{out}}{\dot{m}}\right)_{ideal} = h_{4s} - h_1 = 478.2 \text{ kJ/kg}$$

Thus, less heat is rejected for the ideal cycle.

c) The thermal efficiency is

$$\eta = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{612.7}{1515.4 - 656.39} = 0.287 \ \ (28.7\%)$$

For the ideal cycle

$$\eta = 1 - \frac{h_{4s} - h_1}{h_3 - h_{2s}} = 1 - \frac{478.2}{1515.42 - 590.47} 0.483 \ (48.3\%)$$

Thus, irreversibilities cause a substantial decrease in thermal efficiency.

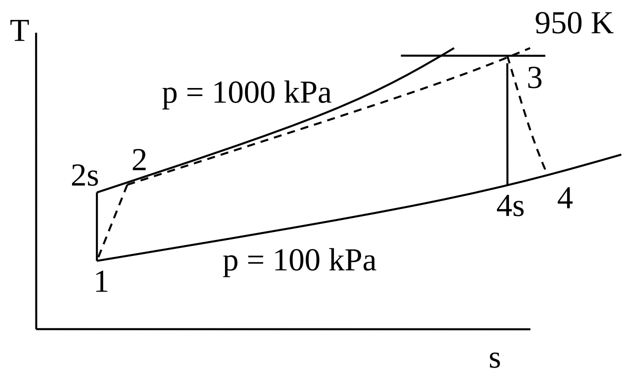
8) Air enters the compressor of a simple gas turbine at 100 kPa, 300 K, with a volumetric flow rate 5 m^3/s. The compressor pressure ratio is 10 and its isentropic efficiency is 85%. At the inlet to the turbine, the pressure is 950 kPa, and temperature is 1400 K. The turbine has an isentropic efficiency of 88% and the exit pressure is 100 kPa. On the basis of an air-standard analysis, evaluate all availability inputs, destructions, and losses. Let T_0 = 300 K, p_0 = 100 kPa.

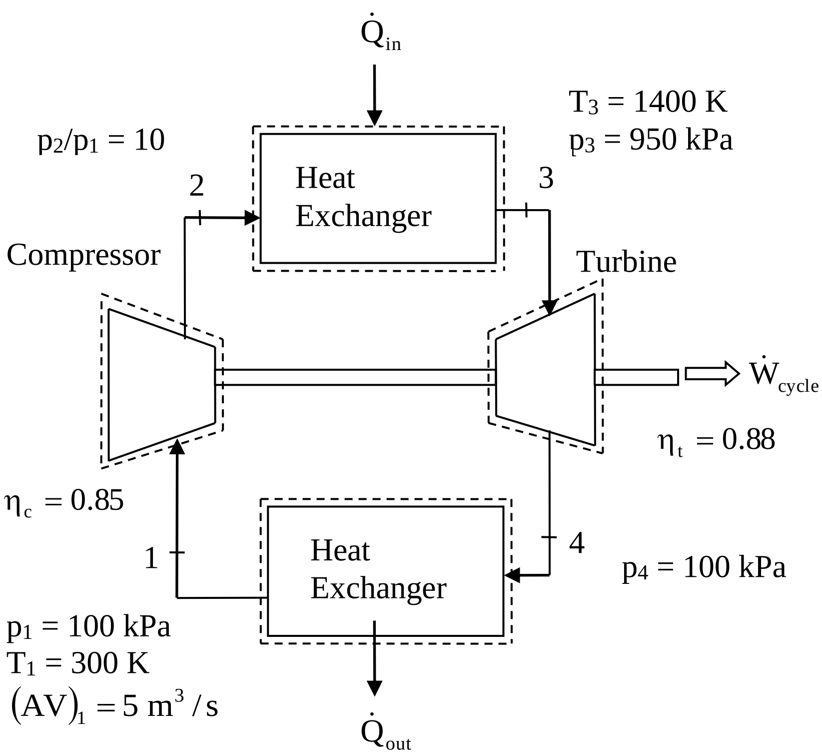
Solution:

Known: A simple gas turbine is analyzed on an air-standard basis from an availability viewpoint. Data are known at various locations.

Find: Determine all availability inputs, destructions, and losses.

Schematic and given data:





Assumptions:

1) Each component is analyzed as a control volume at steady state.
2) The compressor and turbine are adiabatic.
3) Kinetic and potential energy effects are negligible.
4) The working fluid is air modeled as an ideal gas.
5) Let $T_0 = 300\text{ K}$ and $p_0 = 100\text{ kPa}$

Analysis: Data are obtained for each principal state from Tables

State	T(K)	P(kPa)	h(kJ/kg)	s^0 (kJ/kg.K)
1	300	100	300.19	1.70203
2	621.1	1000	629.21	2.44539
3	1400	950	1515.42	3.36200
4	873.9	100	903.72	2.81558

The increase in flow availability of the air passing through the heat exchanger is taken as the net input of availability to the gas turbine.

Input: $\dot{m}(e_{f3} - e_{f2}) = \dot{m}\left[(h_3 - h_2) - T_0(s_3^{\,0} - s_2^{\,0} - R\ln p_3/p_2\right]$

Evaluating $\dot{m}$:

$$\dot{m} = \frac{(AV)_1 p_1}{RT_1} = \frac{(5\text{ m}^3/\text{s})(100\text{ kPa})}{\left(\dfrac{8.314\text{ kJ}}{28.97\text{ kg.K}}\right)(300\text{K})}\left(\frac{10^3\text{ N/m}^2}{1\text{ kPa}}\right)\left(\frac{1\text{ kJ}}{10^3\text{ Nm}}\right) = 5.807\text{ kg/s}$$

$$\dot{m}(e_{f3} - e_{f2}) = \left(5.807\ \frac{\text{kg}}{\text{s}}\right)\left[(1515.42 - 629.21)\frac{\text{kJ}}{\text{kg}} - (300\text{K})\left(3.36200 - 2.44539 - \frac{8.314}{28.97}\ln\frac{950}{1000}\right)\frac{\text{kJ}}{\text{kgK}}\right]$$

$$\times\left(\frac{1\text{kW}}{1\text{kJ/s}}\right)$$

$$\dot{m}(e_{f3} - e_{f2}) = 3524\text{ kW (Input)}$$

Availability is destroyed due to irreversibilities in the compressor and turbine. Thus

Destructions:

$$\dot{I}_{comp} = T_0\dot{m}(s_2 - s_1) = T_0\dot{m}\left[(s_2^{\,0} - s_1^{\,0}) - R\ln p_2/p_1\right] = 143.8\text{ kW}$$

$$\dot{I}_{turb} = T_0\dot{m}\left[(s_4^{\,0} - s_3^{\,0}) - R\ln p_4/p_3\right] = 173.6\text{ kW}$$

The net power developed by the cycle represents the output of availability from the cycle, or

$$\dot{W}_{cycle} = \dot{m}\left[(h_3 - h_4) - (h_2 - h_1)\right] = 1641.5\text{ kW (Output)}$$

Finally, the air enters the gas turbine at p_0 and T_0. Thus, the change in flow availability from inlet to exit represents the loss due to the hot air being discharged. Thus,

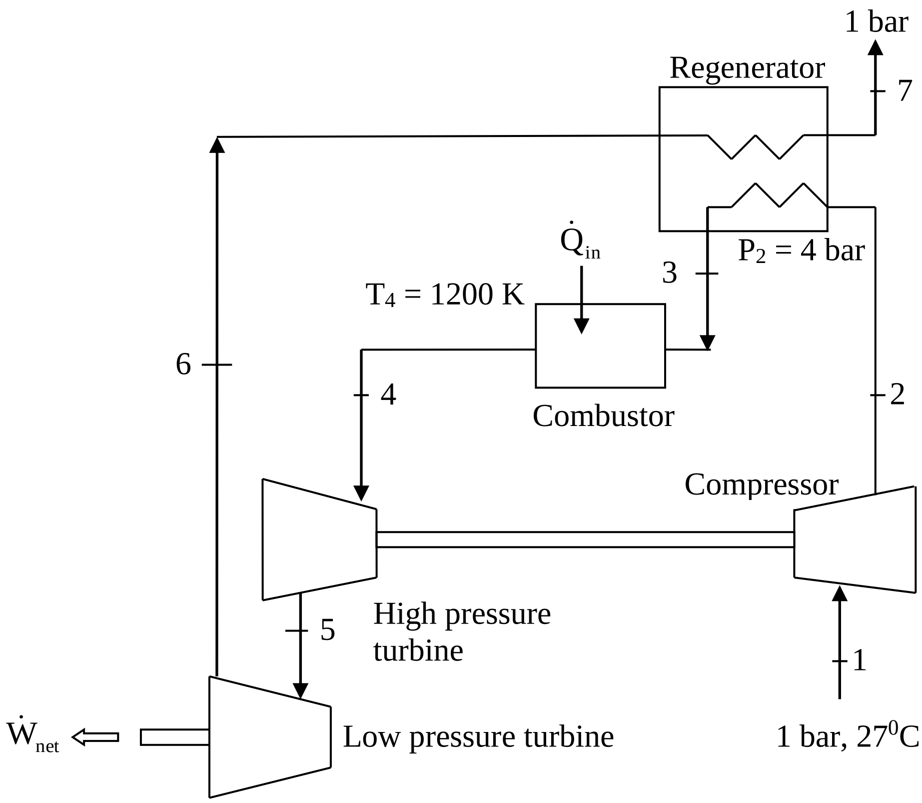
$$\text{Loss}: \dot{m}(e_{f4} - e_{f1}) = \dot{m}\left[(h_4 - h_1) - T_0(s_4^0 - s_1^0 - R\ln p_4/p_1)\right] = 1564.8 \text{ kW}$$

Summarizing

Input: 3524 kW
Disposition
Output: 1641.5 kW
Destroyed: 317.4 Kw
Loss: 1564.8 kW
3523.7 kW

9) A regenerative gas turbine power plant is shown in Figure. Air enters the compressor at 1 bar, 27^0C and is compressed to 4 bars. The isentropic efficiency of the compressor is 80%, and the regenerator effectiveness is 90%. All the power developed by the higher-pressure turbine is used to run the compressor and the lower-pressure turbine provides the net power output of 97 kW. Each turbine has an isentropic efficiency of 87% and the temperature at the inlet to the high-pressure turbine is 1200 K. Determine

a) The mass flow rate of air into the compressor, in kg/s.
b) The thermal efficiency
c) The temperature of the air at the exit of the regenerator, in K.

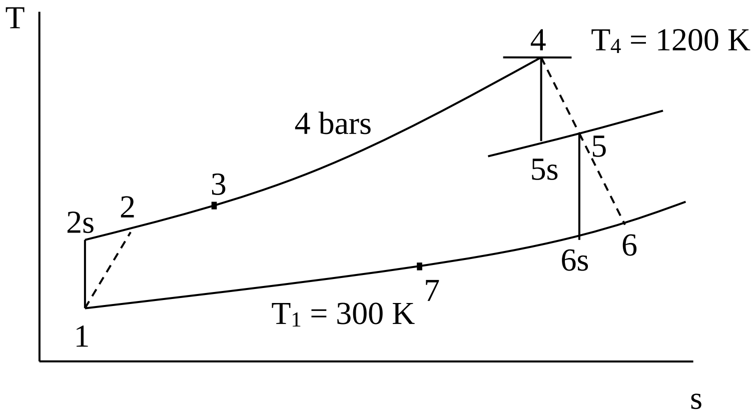


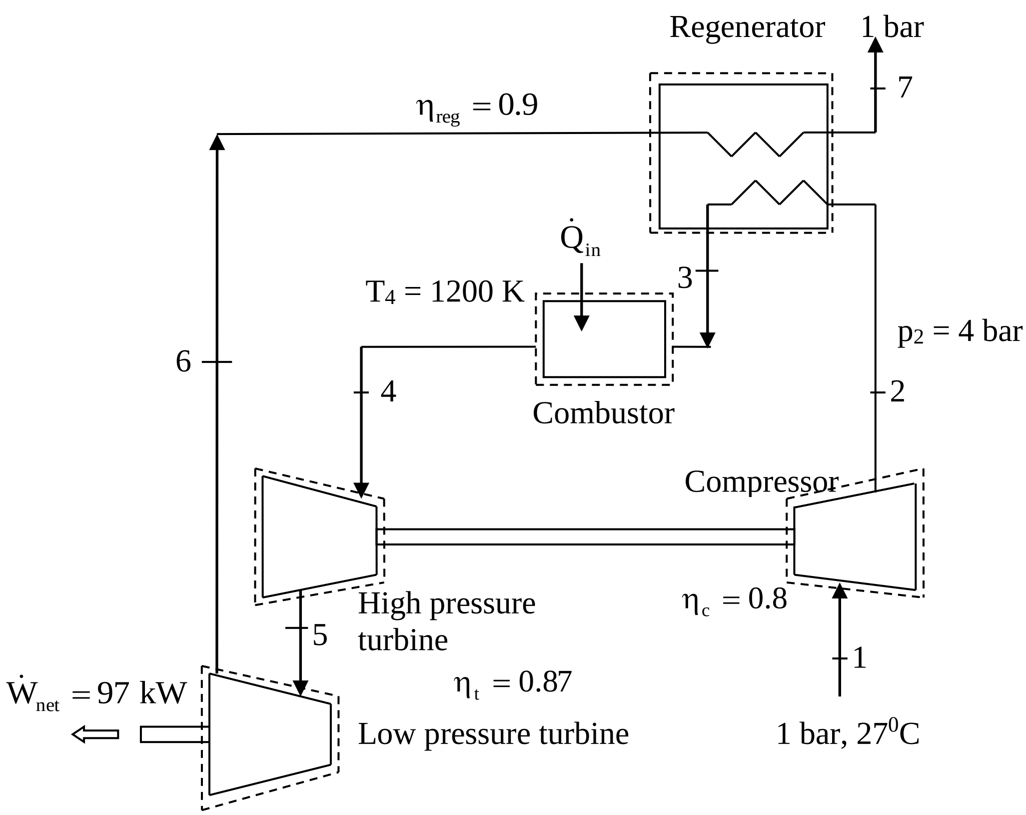
Solution:

Known: In a regenerative gas turbine power plant, a high pressure turbine runs the compressor and the net power output is provided by a low pressure turbine. Data are known at various locations.

Find: Determine (a) the mass flow of air into the compressor, (b) the thermal efficiency, and (c) the temperature of the air at the exit of the regenerator.

Schematic and Given Data:





Assumptions:

1) Each component is analyzed as a control volume at steady state.
2) The compressors, turbines, and regenerator are adiabatic.
3) There are no pressure drops for flow through the heat exchangers.
4) Kinetic and potential energy effects are negligible.
5) The working fluid is air modeled as an ideal gas.

Analysis: First, fix each of the principal states.

State 1:

$T_1 = 300\ \mathrm{K},\ h_1 = 300.19\ \mathrm{kJ/kg},\ pr_1 = 1.3860$

State 2:
For an isentropic compression, $pr_{2s} = (p_2 / p_1)pr_1 = 5.544 \Rightarrow h_{2s} = 446.49\ \mathrm{kJ/kg}$
Using the compressor efficiency,

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 483.06\ \mathrm{kJ/kg}$$

State 4:

$T_4 = 1200\ \mathrm{K} \Rightarrow h_4 = 1277.79\ \mathrm{kJ/kg},\ pr_4 = 238$

State 5:
The work of the compressor and the high pressure turbine are equal. Thus,
$h_2 - h_1 = h_4 - h_5 \Rightarrow h_5 = 1094.92\ \mathrm{kJ/kg}$

From the turbine efficiency; $h_4 - h_5 = \eta_t (h_4 - h_{5s}) \Rightarrow h_{5s} = 1067.59\ \mathrm{kJ/kg}$

Thus, $p_{r5s} = 122.86$ and $p_5 = (p_{r5s} / p_{r4})p_4 = 2.065$ bars

From $h_5 = 1094.92\ \mathrm{kJ/kg} \Rightarrow p_{r5} = 134.714$

State 6: For isentropic expansion through the low pressure turbine

$$p_{r6s} = (p_6 / p_5)p_{r5} = (1/2.065)(134.714) = 65.237 \Rightarrow h_{6s} = 896.40\ \mathrm{kJ/kg}$$

Using the turbine efficiency

$$h_6 = h_5 - \eta_t (h_5 - h_{6s}) = 922.21\ \mathrm{kJ/kg}$$

State 3: Now using the regenerator effectiveness

$$\eta_{reg} = \frac{h_3 - h_2}{h_6 - h_2} \Rightarrow h_3 = \eta_{reg}(h_6 - h_2) + h_2 = 878.3\ \mathrm{kJ/kg}$$

a) the mass flow rate is the same for each component. Thus, for the low pressure turbine

$$\dot{W}_{net} = \dot{m}(h_5 - h_6)$$

or

$$\dot{m} = \frac{\dot{W}_{net}}{(h_5 - h_6)} = \frac{97 \text{ kW}}{(1094.92 - 922.21)\text{ kJ/kg}}\left(\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right) = 0.562 \text{ kg/s}$$

b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{net}}{\dot{m}(h_4 - h_3)} = \frac{97}{(0.562)(1277.79 - 878.3)} = 0.432 \text{ (43.2\%)}$$

c) The specific enthalpy h_7 is found from an energy balance for the regenerator,

$$0 = (h_2 - h_3) + (h_6 - h_7)$$

or

$$h_7 = (h_2 - h_3) + h_6 = 483.06\frac{\text{kJ}}{\text{kg}} - 878.3\frac{\text{kJ}}{\text{kg}} + 922.21\frac{\text{kJ}}{\text{kg}} = 526.97 \text{ kJ/kg}$$

Thus, from Table; $T_7 = 523.2 \text{ K}$

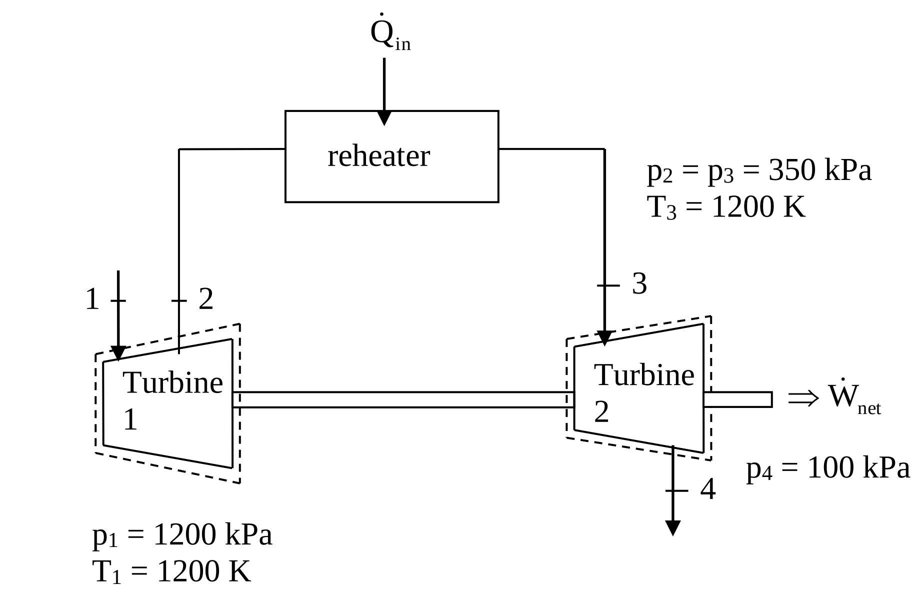
10) Air enters the turbine of a gas turbine at 1200 kPa, 1200 K, and expands to 100 kPa in two stages. Between the stages, the air is reheated at a constant pressure of 350 kPa to 1200 K. The expansion through each turbine stage is isentropic. Determine, in kJ per kg of air flowing

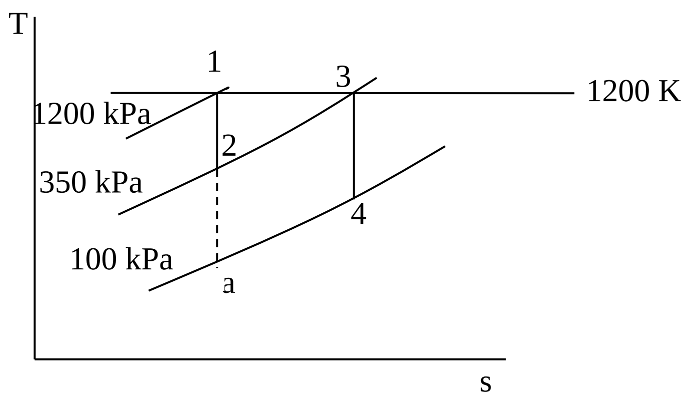
a) The work developed by each stage
b) The heat transfer for the reheat process
c) The increase in net work as compared to a single stage of expansion with no reheats.

Solution:

Known: Air expands in two stages through a turbine with reheat between the stages. The states are specified at the inlet and exit of each component.
Find: Determine per unit mass of air flowing, (a) the work developed by each stage, (b) the heat transfer for reheat, and (c) the increase in net work compared to a single stage of expansion with no reheat.

Schematic and Given Data:





Assumptions:

1) Each component volume is at steady state.
2) The turbines operate isentropically.
3) Kinetic and potential energy effects are negligible.
4) The working fluid is air modeled as an ideal gas.

Analysis: First, fix each of the principal states.

State 1: $T_1 = 1200 \text{ K} \Rightarrow h_1 = 1277.79 \text{ kJ/kg}, \ p_{r1} = 238.0$

State 1: $p_{r2} = (p_2 / p_1) p_{r1} = 69.417 \Rightarrow h_2 = 912.11 \text{ kJ/kg}$

State 3: $T_3 = 1200 \text{ K} \Rightarrow h_3 = h_1 = 1277.79 \text{ kJ/kg}, \ p_{r3} = p_{r1} = 238.0$

State 4: $p_{r4} = (p_4 / p_3) p_{r3} = 68 \Rightarrow h_4 = 906.85 \text{ kJ/kg}$

a) The work developed by each stage is

$$\dot{W}_{t1} / \dot{m} = h_1 - h_2 = 365.68 \text{ kJ/kg}$$
$$\dot{W}_{t2} / \dot{m} = h_3 - h_4 = 370.94 \text{ kJ/kg}$$

b) For the reheater

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 = (1277.79 - 912.11) = 365.7 \text{ kJ/kg}$$

c) To determine the work for a single stage of expansion, determine h_a, as follows.

$$Pr_a = (P_a / P_1) Pr_1 = 19.833 \Rightarrow h_a = 638.58 \text{ kJ/kg}$$

Thus $\dot{W} / \dot{m} = (h_1 - h_a) = 639.21 \text{ kJ/kg}$

and

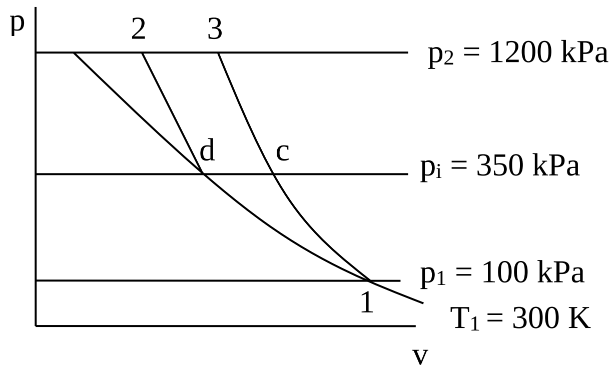
$$\% \text{ increase } = \frac{(365.68 + 370.94) - 639.21}{639.2} \times 100 = 15.2\ \%$$

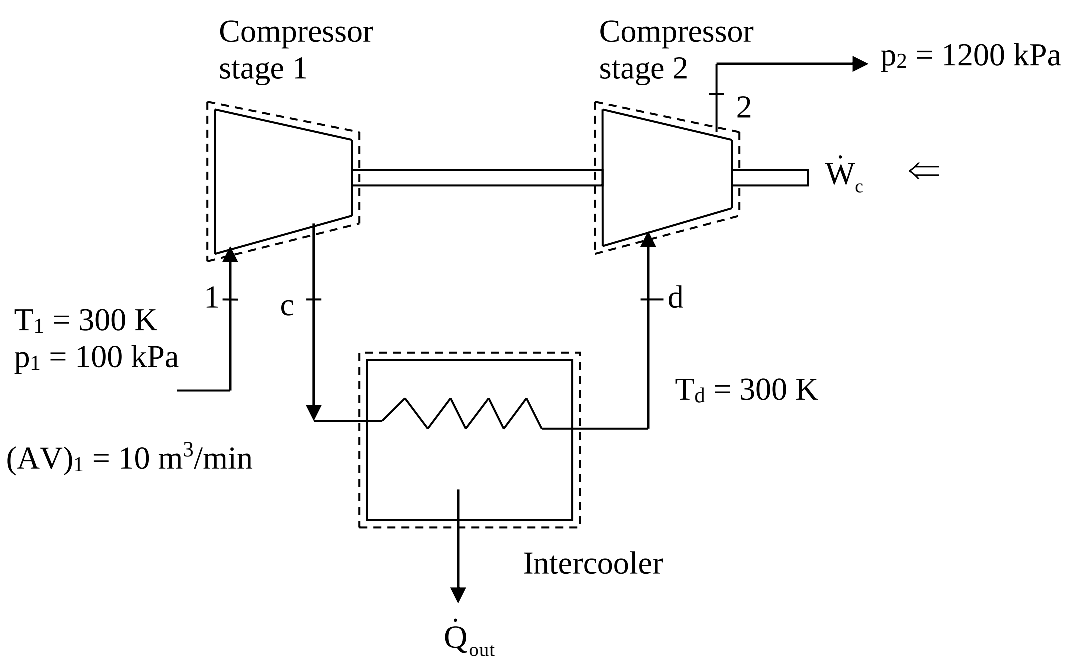
11) A two-stage air compressor operates at steady state, compressing $10 m^3/min$ of air from 100 kPa, 300 K, to 1200 kPa. An intercooler between the two stages cools the air to 300 K at a constant pressure of 350 kPa. The compression processes are isentropic. Calculate the power required to run the compressor, in kW, and compare the result to the power required for isentropic compression from the same inlet state to the same final pressure.

Solution:

Known: Air is compressed into a two-stage compressor with intercooling between the stages. Operating pressures and temperatures are given.

Find: Determine the power to run the compressor and compare this to the power required for isentropic compression from the same inlet state to the same final pressure.
Schematic and Given Data:





Assumptions:

1) The compressor stages and intercooler are analyzed as control volumes at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2) The compression processes are isentropic.
3) There is no pressure drop for flow through the intercooler. Kinetic and potential energy effects are negligible. The air is modeled as an ideal gas.

Analysis: First, fix each of the principal states.

State 1: $T_1 = 300\ \text{K} \Rightarrow h_1 = 300.19\ \text{kJ/kg},\ p_{r1} = 1.3860$
State c: $p_{rc} = (p_c/p_1)p_{r1} = 4.851 \Rightarrow h_c = 429.77\ \text{kJ/kg}$
State d: $T_d = T_1 = 300\ \text{K} \Rightarrow h_d = 300.19\ \text{kJ/kg},\ p_{rd} = 1.3860$
State 2: $p_{r2} = (p_2/p_d)p_{rd} = 4.752 \Rightarrow h_2 = 427.21\ \text{kJ/kg}$
The mass flow rate is

$$\dot{m} = \frac{(AV)_1 p_1}{RT_1} = \frac{(10\ \text{m}^3/\text{min})(100\,\text{kPa})}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kgK}}\right)(300\,\text{K})}\left(\frac{1\ \text{min}}{60\,\text{s}}\right)\left(\frac{10^3\ \text{N/m}^2}{1\ \text{kPa}}\right)\left(\frac{1\ \text{kJ}}{10^3\ \text{Nm}}\right) = 0.1936\ \text{kg/s}$$

The compressor power is

$$\dot{W}_c = \dot{W}_{c1} + \dot{W}_{c2} = \dot{m}(h_c - h_1) + \dot{m}(h_2 - h_d)$$

$$\dot{W}_c = \left(0.1936\ \frac{\text{kg}}{\text{s}}\right)(429.77 - 300.19)\frac{\text{kJ}}{\text{kg}}\left(\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right) + (0.1936)(427.21 - 300.19)$$

$$\dot{W}_c = 49.68\ \text{kW}$$

To find the power a single stage of compression, determine h_3 as follows:

$$p_{r3} = (p_2/p_1)p_{r1} = 16.632 \Rightarrow h_3 = 610.65\ \text{kJ/kg}$$

Thus, $\dot{W} = \dot{m}(h_3 - h_1) = 60.11\ \text{kW}$

The decrease in power with two-stage, intercooled compression is

$$\%\ \text{decrease} = \frac{60.11 - 49.68}{60.11} \times 100 = 17.35\%$$

12) Air at 22 kPa, and 220 K, and 250 m/s enters a turbojet engine in flight at an altitude of 10,000 m. The pressure ratio across the compressor is 12. The turbine inlet temperature is 1400 K, and the pressure at the nozzle exit is 22 kPa. The diffuser and nozzle processes are isentropic efficiencies of 85 and 88%, respectively, and there is no pressure drop for flow through the combustor. On the basis of an air-standard analysis, determine

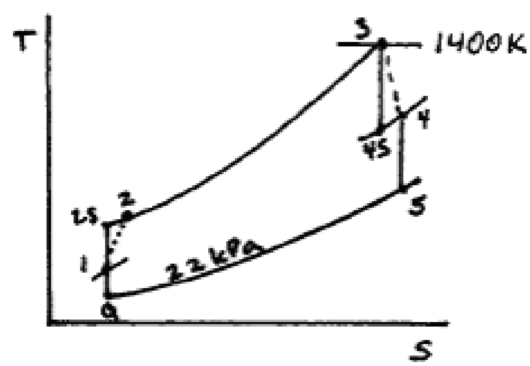
a) The pressures and temperatures at each principal state, in kPa and K, respectively.
b) The velocity at the nozzle exit, in m/s

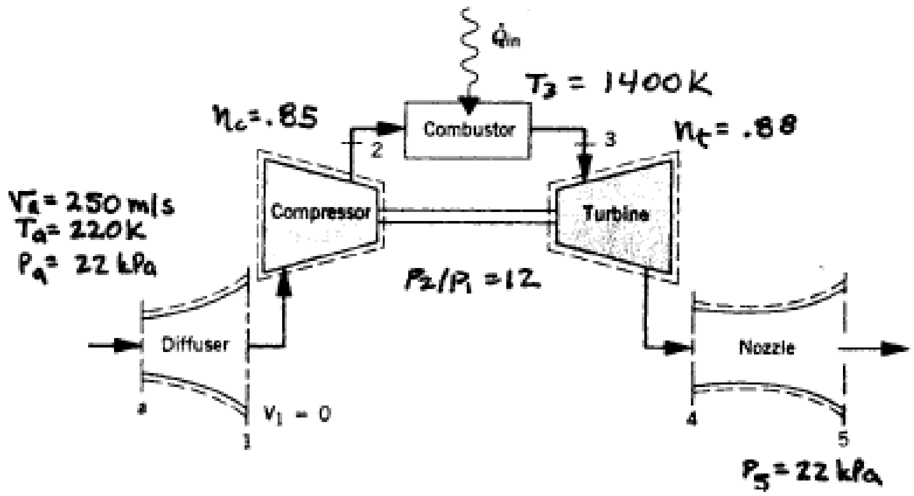
Neglect kinetic energy except at the diffuser inlet and the nozzle exit.

Solution:

Known: A turbojet engine is analyzed as an air standard basis. Data are known at various locations.
Find: Determine (a) the pressures and temperatures at each principal state and (b) the velocity at the nozzle exit.

Schematic and Given Data:





Assumptions:

1) Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2) There is no pressure drop for flow through the combustor.
3) The turbine work output is just sufficient to drive the compressor.
4) Except at the inlet and exit of the engine, kinetic energy effects can be ignored. Potential energy effects are negligible throughout.
5) The working fluid is air modeled as an ideal gas.

Analysis: First, fix each of the principal states.

State a: $T_a = 220\text{ K},\ p_a = 22\text{ kPa} \Rightarrow h_a = 219.97\text{ kJ/kg},\ p_{ra} = 0.4690$

State 1: An energy balance for the diffuser gives; $0 = h_a + \frac{V_a^2}{2} - h_1$. Thus,

$$h_1 = h_a + \frac{V_a^2}{2} = 219.97\,\frac{\text{kJ}}{\text{kg}} + \frac{250^2\,\text{m}^2/\text{s}^2}{2}\left(\frac{1\text{N}}{1\text{kgm}/\text{s}^2}\right)\left(\frac{1\text{ kJ}}{10^3\text{ Nm}}\right) = 251.22\text{ kJ/kg} \Rightarrow p_{r1} = 0.7454$$

Since the process is isentropic, $p_1 = (p_{r1}/p_{ra})p_a = 34.97\text{ kPa}$

State 2: For isentropic compression, $p_{r2} = (p_2/p_1)p_{r1} = 8.9453$

$h_{2s} = 511.9\text{ kJ/kg}.$ With the compressor efficiency

$$h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 557.9\text{ kJ/kg},\ p_2 = 12\text{ kPa},\ p_1 = 419.64\text{ kPa}$$

State 3: $T_3 = 1400\text{ K},\ p_3 = p_2 \Rightarrow h_3 = 1515.42\text{ kJ/kg},\ p_{r3} = 450.5$

State 4: For a turbojet, $\dot{W}_c / \dot{m} = \dot{W}_t / \dot{m}$. Thus

$(h_2 - h_1) = \eta_t (h_3 - h_{4s}) \Rightarrow h_{4s} = 1166.92 \text{ kJ/kg}$

Thus, $p_{r4s} = 170.28$ and $p_4 = p_3 (p_{r4s} / p_{r3}) = 158.62 \text{ kPa}$

And

$h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 1208.74 \text{ kJ/kg} \Rightarrow p_{r4} = 193.81$

State 5:

$p_{r5} = p_{r4} (p_5 / p_4) = 26.881; \quad h_5 = 699.48 \text{ kJ/kg}$

b) An energy balance on the nozzle gives

$$0 = h_4 - \left(h_5 + \frac{V_5^2}{2} \right)$$

or

$$V_5 = \sqrt{2(h_4 - h_5)} = \sqrt{2(1208.74 - 699.48) \frac{\text{kJ}}{\text{kg}} \left(\frac{10^3 \text{ Nm}}{1 \text{ kJ}} \right) \left(\frac{1 \text{ kgm/s}^2}{1 \text{ N}} \right)}$$

$$= 1009 \text{ m/s}$$

13) A combined gas turbine-vapor power plant has a net power output of 10 MW. Air enters the compressor of the gas turbine at 100 kPa, 300 K, and is compressed to 1200 kPa. The isentropic efficiency of the compressor is 84%. The conditions at the inlet to the turbine are 1200 kPa and 1400 K. Air expands through the turbine, which has an isentropic efficiency of 88%, to a pressure of 100 kPa. The air then passes through the interconnecting heat exchanger, and is finally discharged at 480 K. Steam enters the turbine of the vapor power cycle at 8 MPa, 400^0C, and expands to the condenser pressure of 8 kPa. Water enters the pump as saturated liquid at 8 kPa. The turbine and pump have isentropic efficiencies of 90 and 80%, respectively. Determine

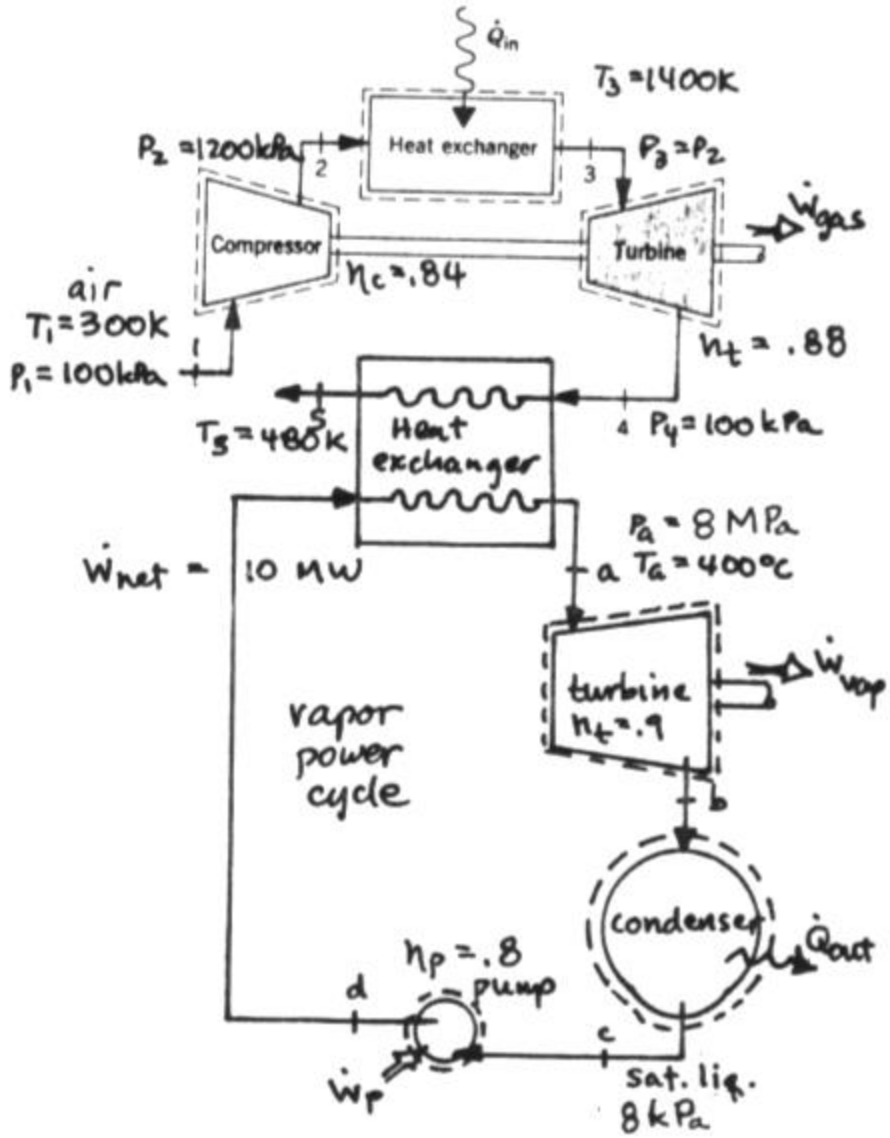
a) The mass flow rates of air and water, each in kg/s.
b) The rate of heat transfer to the combined cycle, in MW.
c) The thermal efficiency of the combined cycle.

Solution:

Known: A combined gas turbine-vapor power plant has a known net power output. Data are known at various locations in both cycles.

Find: Determine (a) The mass flow rates of water and air, (b) the rate of heat transfer to the combined cycle, and (c) the overall thermal efficiency.

Schematic and Given Data:



Assumptions:

1) Each component is analyzed as a control volume at steady state.
2) The gas turbine is analyzed on an air-standard basis.
3) The turbines, compressor, pump, and interconnecting heat exchanger operate adiabatically.
4) Kinetic and potential energy effects are negligible.
5) The working fluid for the gas turbine is air modeled as an ideal gas.

Analysis: First, fix each of the principal states. For the gas turbine cycle:

State 1: $T_1 = 300 \text{ K} \Rightarrow h_1 = 300.19 \text{ kJ/kg},\ p_{r1} = 1.3860$

State 2: For isentropic compression,

$$p_{r2} = (p_2 / p_1) p_{r1} = 16.632 \Rightarrow h_{2s} = 610.65 \text{ kJ/kg}$$

Using the compressor efficiency,

$$h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 669.78\ kJ/kg$$

State 3: $T_3 = 1400\ K \Rightarrow h_3 = 1515.42\ kJ/kg,\ p_{r3} = 450.5$

State 4: $p_{r4} = (p_4/p_3)p_{r3} = 37.542 \Rightarrow h_{4s} = 768.38\ kJ/kg,\ \eta_t = 0.88 \Rightarrow h_4 = 858\ kJ/kg$

State 5: $T_5 = 480\ K \Rightarrow h_5 = 482.49\ kJ/kg$

For the vapor cycle;

State a: $T_a = 400^0 C,\ p_a = 8\ MPa \Rightarrow h_a = 3138.3\ kJ/kg,\ s_a = 6.3634\ kJ/kgK$

State b: $p_b = 8\ kPa,\ s_{bs} = s_a \Rightarrow x_{bs} = 0.7557,\ h_{bs} = 1989.9\ kJ/kg$

Thus, with the turbine efficiency, $h_b = h_a - \eta_t(h_a - h_{bs}) = 2104.74\ kJ/kg$

State c: $p_c = 8\ kPa$, saturated liquid $\Rightarrow h_c = 173.88\ kJ/kg$

State d:

$$h_{ds} = h_c + v_c(p_d - p_c)$$

$$h_{ds} = 173.88\frac{kJ}{kg} + \left(1.0084\times10^{-3}\ \frac{m^3}{kg}\right)(80-0.08)bars\left(\frac{10^5\ N/m^2}{1\ bar}\right)\left(\frac{1\ kJ}{10^3\ Nm}\right)$$

$$h_{ds} = 181.94\ kJ/kg$$

$$h_d = h_c + \frac{h_{ds} - h_c}{\eta_p} = 183.96\ kJ/kg$$

a) To find the mass flow rates of water $(\dot{m}_{st})$ and air $(\dot{m}_{air})$, begin with energy and mass balances on the interconnecting heat exchanger

$$0 = \dot{m}_{air}(h_4 - h_5) + \dot{m}_{st}(h_d - h_a)$$

or

$$\frac{\dot{m}_{st}}{\dot{m}_{air}} = \frac{h_4 - h_5}{h_a - h_d} = \frac{858\ (kJ/kg) - 482.49\ (kJ/kg)}{3138.3\ (kJ/kg) - 183.96\ (kJ/kg)} = 0.1217$$

For the gas turbine cycle

$$\dot{W}_{gas} = \dot{m}_{air} = [(h_3 - h_4) - (h_2 - h_1)] = \dot{m}_{air}(287.83\ kJ/kg)$$

and for the vapor cycle

$$\dot{W}_{vap} = \dot{m}_{st}\left[(h_a - h_b) - (h_d - h_c)\right] = \dot{m}_{st}(1023.5)$$

$$\text{Thus, } \dot{W}_{net} = \dot{m}_{air}\left[(287.83) + \frac{\dot{m}_{st}}{\dot{m}_{air}}(1023.5)\right]$$

$$\dot{m}_{air} = \frac{10000 \text{ kJ/s}}{(287.83)\frac{\text{kJ}}{\text{kg}} + (0.1271)(1023.5)\frac{\text{kJ}}{\text{kg}}} = 23.93 \text{ kg/s}$$

$$\dot{m}_{st} = (0.1271)\dot{m}_{air} = 3.041 \text{ kg/s}$$

b) The rate of heat addition to the combined cycle is

$$\dot{Q}_{in} = \dot{m}_{air}(h_3 - h_2) = \left(23.93 \frac{\text{kg}}{\text{s}}\right)(1515.42 - 669.78)\frac{\text{kJ}}{\text{kg}}\left(\frac{1 \text{ MW}}{10^3 \text{ kJ/s}}\right)$$

$$\dot{Q}_{in} = 20.24 \text{ MW}$$

c) The overall thermal efficiency is

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{10}{20.24} = 0.494 \ \ (49.4\%)$$

14) Air enters the compressor of an Ericsson cycle at 300 K, 1 bar, with a mass flow rate of 5 kg/s. The pressure and temperature at the inlet to the turbine are 10 bar and 1400 K, respectively. Determine

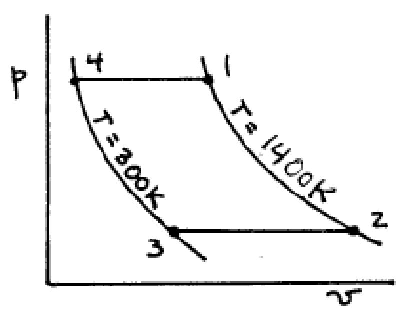
a) The net power developed, in kW.
b) The thermal efficiency.
c) The back work ratio.

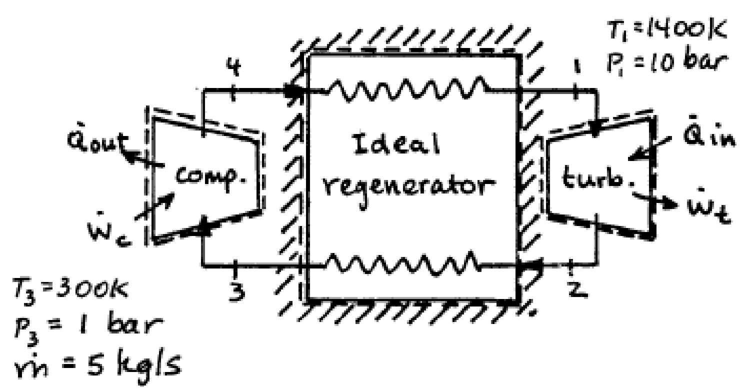
Solution:

Known: Air is the working fluid in an Ericsson cycle with data known at various locations.

Find: Determine a) the net power developed, b) the thermal efficiency, c) the backwork ratio

Schematic and Given Data:





Assumptions:

1) Each component is analyzed as a control volume at steady state.
2) All processes are internally reversible.
3) The compression and expansion processes are isothermal.
4) Kinetic and potential energy effects are negligible.
5) The air behaves as an ideal gas.

Analysis: a) The turbine power is evaluated using

$$\dot{W}_t = -\dot{m}\int_1^2 v\,dp = -\dot{m}RT_1 \ln(p_2/p_1)$$

$$\dot{W}_t = -(5\,\text{kg/s})\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kgK}}\right)(1400\ \text{K})\ln\left(\frac{1}{10}\right)\left(\frac{1\,\text{kW}}{1\ \text{kJ/s}}\right) = 4626\ \text{kW}$$

and for the compressor

$$\dot{W}_c = \dot{m}\int_3^4 v dp = \dot{m}RT_3 \ln(p_4/p_3)$$

$$\dot{W}_c = (5\,kg/s)\left(\frac{8.314}{28.97}\frac{kJ}{kgK}\right)(300\ K)\ln\left(\frac{10}{1}\right)\left(\frac{1 kW}{1\ kJ/s}\right) = 991.2\ kW$$

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = 3635\ kW$$

b) The thermal efficiency is

$$\eta = 1 - \frac{T_3}{T_1} = 1 - \frac{300}{1400} = 0.786\ \ (78.6\%)$$

Alternatively, from an energy balance on the turbine;

$$\dot{Q}_{in} = \dot{W}_t$$

Thus,

$$\eta = \frac{\dot{W}_{cycle}}{Q_{in}} = \frac{3635}{4626} = 0.786$$

c) The backwork ratio is

$$bwr = \frac{\dot{W}_c}{\dot{W}_t} = \frac{991.2}{4626} = 0.214$$

15) Consider an ideal gas-turbine cycle with two stages of compression and two stages of expansion. The pressure ratio across each stage of compressor and turbine is 3. The air enters each stage of the compressor at 300 K and each stage of the turbine at 1200 K. Determine the back work ratio and the thermal efficiency of the cycle, assuming

a) no regenerator is used and
b) a regenerator with 75 percent effectiveness is used. Use constant specific heats at room temperature

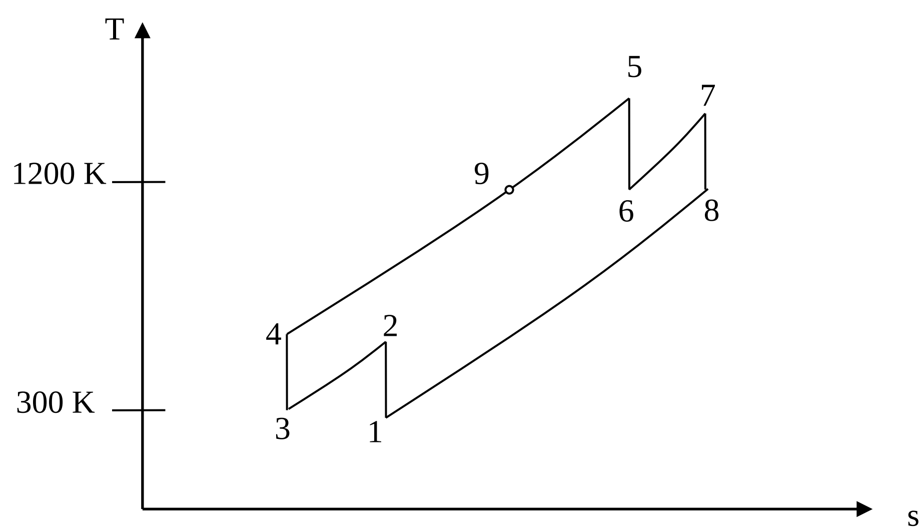
Solution:

Assumptions:

1) The air standard assumptions are applicable.
2) Air is an ideal gas with variable specific heats.
3) Kinetic and potential energy changes are negligible.

Analysis: a) The work inputs to each stage of compressor are identical, so are the work outputs of each stage of the turbine since this is an ideal cycle. Then

$$T_1 = 300 \text{ K} \Rightarrow h_1 = 300.19 \text{ kJ/kg}$$
$$p_{r1} = 1.386$$
$$p_{r2} = \frac{p_2}{p_1} p_{r1} = (3)(1.386) \Rightarrow h_2 = h_4 = 411.26 \text{ kJ/kg}$$
$$T_5 = 1200 \text{ K} \Rightarrow h_5 = h_7 = 1277.79 \text{ kJ/kg}$$
$$p_{r5} = 238$$



$$p_{r6} = \frac{p_6}{p_5} p_{r5} = \left(\frac{1}{3}\right)(238) = 79.33 \Rightarrow h_6 = h_8 = 946.36 \text{ kJ/kg}$$

$$W_{c,in} = 2(h_2 - h_1) = 2(411.26 - 300.19) = 222.14 \text{ kJ/kg}$$

$$W_{T,out} = 2(h_5 - h_6) = 2(1277.79 - 946.36) = 662.86 \text{ kJ/kg}$$

Thus ,

$$r_{bw} = \frac{W_{c,in}}{W_{T,out}} = \frac{222.14 \text{ kJ/kg}}{662.86 \text{ kJ/kg}} = 33.5\ \%$$

$$q_{in} = (h_5 - h_4) + (h_7 - h_6) = (1277.79 - 411.26) + (1277.79 - 946.36) = 1197.96 \text{ kJ/kg}$$

$$W_{net} = W_{Tout} - W_{c,in} = 662.86 - 222.14 = 440.72 \text{ kJ/kg}$$

$$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{440.72 \text{ kJ/kg}}{1197.96 \text{ kJ/kg}} = 36.8\%$$

b) When a regenerator is used, r_{bw} remains the same. The thermal efficiency in this case becomes

$$q_{regen} = \eta_{reg}(h_9 - h_4) = (0.75)(946.36 - 411.26)\frac{\text{kJ}}{\text{kg}} = 401.33 \text{ kJ/kg}$$

$$q_{in} = q_{in,old} - q_{regen} = 1197.96\frac{\text{kJ}}{\text{kg}} - 401.33\frac{\text{kJ}}{\text{kg}} = 796.63 \text{ kJ/kg}$$

$$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{440.72 \text{ kJ/kg}}{796.63 \text{ kJ/kg}} = 55.3\ \%$$