CANKAYA UNIVERSITY
FACULTY OF ENGINEERING AND ARCHITECTURE
MECHANICAL ENGINEERING DEPARTMENT
ME 212 THERMODYNAMICS II

CHAPTER 10

EXAMPLES SOLUTION

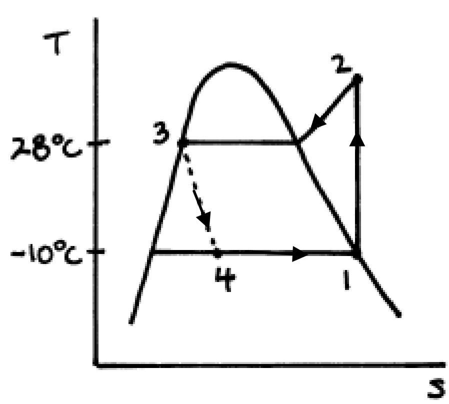
1) An ideal vapor-compression refrigerant cycle operates at steady state with Refrigerant 134a as the working fluid. Saturated vapor enters the compressor at -10^0C, and saturated liquid leaves the condenser at 28^0C. The mass flow rate of refrigerant is 5 kg/min. Determine

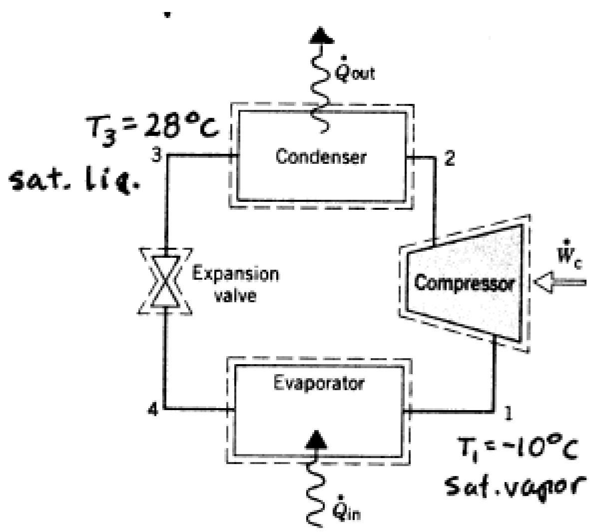
(a) The compressor power, in kW
(b) The refrigerating capacity, in tons.
(c) The coefficient of performance.

Solution:

Known: Refrigerant 134a is the working fluid in an ideal vapor-compression refrigerant cycle. Operating data are known.

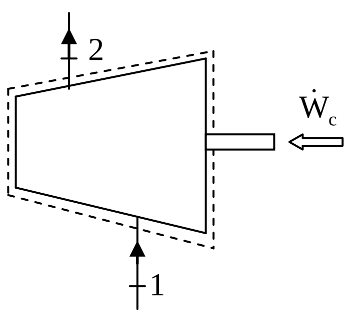
Schematic & Given Data:





Analysis: First, fix each of the principle states.

Compressor:



Assumptions:

- steady state steady flow process(SSSF)
- open system
- $\Delta KE = \Delta PE = 0$
- Neglect heat transfer (q=0)

$$\frac{dE_{c.v}}{dt} = \dot{Q}_{c.v.} - \dot{W}_{c.v.} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

$$0 = \dot{W}_c + \dot{m}_1 h_1 - \dot{m}_2 h_2$$

$$\sum \dot{m}_i = \sum \dot{m}_e$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\dot{W}_c = \dot{m}(h_2 - h_1)$$

$$\frac{dS_{cv}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$$

$$0 = \dot{m}_1 s_1 - \dot{m}_2 s_2$$

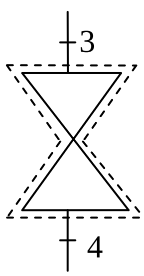
$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$s_2 = s_1$$

State 1: T_1 = -10^{o}C, saturated vapor $\Rightarrow$ h_1 = 241.35 kJ/kg, s_1 = 0.9253 kJ/kg.K

State 2: $P_2 = P_{sat@28^oC}$=7.2675 bars, $s_2 = s_1$ $\Rightarrow$ Double interpolation gives: h_2 = 267.9 kJ/kg

Expansion valve:



Assumptions:

- steady state steady flow process(SSSF)
- open system
- $\Delta KE = \Delta PE = 0$
- Neglect heat transfer (q=0)
- Neglect work (w=0)

$$\frac{dE_{c.v}}{dt} = \underbrace{\dot{Q}_{c.v.}}_{0} - \underbrace{\dot{W}_{c.v.}}_{0} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

$$0 = \dot{m}_3 h_3 - \dot{m}_4 h_4$$

$$\sum \dot{m}_i = \sum \dot{m}_e$$

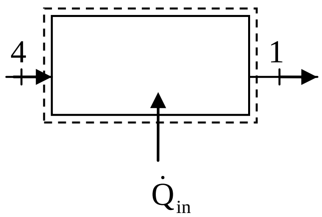
$$\dot{m}_3 = \dot{m}_4 = \dot{m}$$

$$h_3 = h_4$$

State 3: T_3 = 28^{o}C, saturated liquid $\Rightarrow$ h_3 = 88.61 kJ/kg

State 4: Throttling process $\Rightarrow$ $h_4 = h_3$ = 88.61 kJ/kg

Evaporator:



Assumptions:

- steady state steady flow process(SSSF)
- open system
- ΔKE = ΔPE =0
- Neglect work (w = 0)

$$\frac{dE_{c.v}}{dt} = \dot{Q}_{c.v.} \underbrace{- \dot{W}_{c.v.}}_{0} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

$$0 = \dot{Q}_{in} + \dot{m}_4 h_4 - \dot{m}_1 h_1$$

$$\sum \dot{m}_i = \sum \dot{m}_e$$

$$\dot{m}_1 = \dot{m}_4 = \dot{m}$$

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4)$$

(a) The compressor power is:

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(5 \frac{kg}{min}\right)\left(\frac{1\,min}{60\,s}\right)(267.9 - 241.35)\frac{kJ}{kg}\left(\frac{1\,kW}{1\,kJ/s}\right) = 2.212\,kW$$

(b) The refrigeration capacity is:

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(5 \frac{kg}{min}\right)(241.35 - 88.61)\frac{kJ}{kg}\left(\frac{1\,ton}{211\,kJ/min}\right) = 3.62\,tons$$

(c) The coefficient of performance is:

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{(241.35 - 88.61)kJ/kg}{(267.9 - 241.35)kJ/kg} = 5.75$$

2) A vapor-compression refrigeration system circulates Refrigerant 134a at rate of 6 kg/min. The refrigerant enters the compressor at -10^0C, 1.4 bar, and exits at 7 bar. The isentropic compressor efficiency is 67%. There are no appreciable pressure drops as the refrigerant flows through the condenser and evaporator. The refrigerant leaves the condenser at 7 bar, 24^0C. Ignoring heat transfer between the compressor and its surroundings, determine

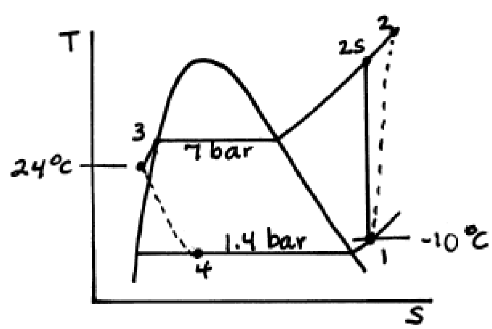
(a) The coefficient of performance.
(b) The refrigerating capacity, in tons.
(c) The irreversibility rates of the compressor and expansion valve, each in kW
(d) The changes in specific flow availability of the refrigerant passing through the evaporator and condenser, respectively, each in kJ/kg.

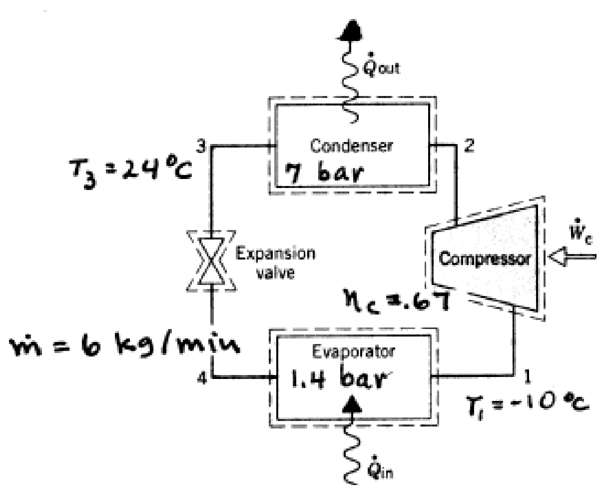
Let $T_o = 21^oC$, $p_o = 1$ bar

Solution:

Known: A vapor-compression refrigeration system circulates Refrigerant 134a with a known mass flow rate. Data are given at various locations, the compressor efficiency is specified.

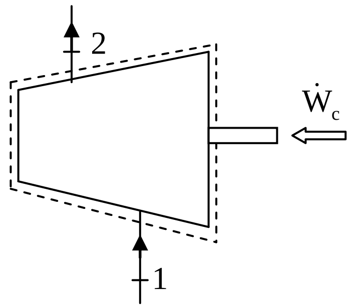
Schematic & Given Data:





Analysis: First, fix each of the principal states.

Compressor:



Assumptions:

- steady state steady flow process(SSSF)
- open system
- ΔKE = ΔPE =0
- Neglect heat transfer (q=0)

$$\frac{dE_{c.v}}{dt} = \dot{Q}_{c.v.} - \dot{W}_{c.v.} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

$$0 = \dot{W}_c + \dot{m}_1 h_1 - \dot{m}_2 h_{2s}$$

$$\sum \dot{m}_i = \sum \dot{m}_e$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\dot{W}_c = \dot{m}(h_{2s} - h_1)$$

$$\frac{dS_{cv}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$$

$$0 = \dot{m}_1 s_1 - \dot{m}_2 s_{2s}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$s_{2s} = s_1$$

State 1: T_1 = -10°C, P_1 = 1.4 bars $\Rightarrow$ h_1 = 243.40 kJ/kg, s_1 = 0.9606 kJ/kg.K

State 2: For isentropic compression, P_2 = 7 bars, $s_{2s} = s_1 \Rightarrow$ h_{2s} = 278.06 kJ/kg

Using the compressor efficiency,

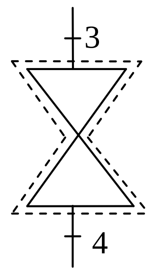
$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 243.40 + \frac{278.06 - 243.40}{0.67} = 295.13\,\text{kJ/kg}$$

s_2 = 1.0135 kJ/kg.K

State 3:

P_3 = 7 bars, T = 24°C $\Rightarrow$ $h_3 = h_{f@24^oC}$ = 82.90 kJ/kg, s_3 = 0.3113 kJ/kg.K

Expansion valve:



Assumptions:

- steady state steady flow process(SSSF)
- open system
- $\Delta KE = \Delta PE = 0$
- Neglect heat transfer (q=0)
- Neglect work (w=0)

$$\frac{dE_{c.v}}{dt} = \underbrace{\dot{Q}_{c.v.}}_{0} - \underbrace{\dot{W}_{c.v.}}_{0} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

$$0 = \dot{m}_3 h_3 - \dot{m}_4 h_4$$

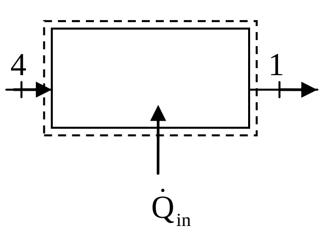
$$\sum \dot{m}_i = \sum \dot{m}_e$$

$$\dot{m}_3 = \dot{m}_4 = \dot{m}$$

$$h_3 = h_4$$

State 4: Throttling process $\Rightarrow$ $h_4 = h_3$ = 82.90 kJ/kg, s_4 = 0.33011 kJ/kg.K

Evaporator:



Assumptions:

- steady state steady flow process(SSSF)
- open system
- $\Delta KE = \Delta PE = 0$
- Neglect work (w = 0)

$$\frac{dE_{c.v}}{dt} = \dot{Q}_{c.v.} \underbrace{-\dot{W}_{c.v.}}_{0} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

$$0 = \dot{Q}_{in} + \dot{m}_4 h_4 - \dot{m}_1 h_1$$

$$\sum \dot{m}_i = \sum \dot{m}_e$$

$$\dot{m}_1 = \dot{m}_4 = \dot{m}$$

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4)$$

(a) The coefficient of performance is:

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 3.10$$

(b) The refrigerating capacity is:

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(6\frac{kg}{min} \right)(243.40 - 82.90)\frac{kJ}{kg}\left(\frac{1\,ton}{211\,kJ/min} \right) = 4.564\,tons$$

(a) For the compressor: $0 = \underbrace{\sum_J \left(\frac{\dot{Q}}{T} \right)_j}_{0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{comp}$

Thus:

$$\dot{I}_{comp} = T_o \dot{\sigma}_{comp} = T_0 \dot{m}(s_2 - s_1) = (294K)\left(6\frac{kg}{min} \right)\left(\frac{1min}{60s} \right)(1.0135 - 0.9606)\left(\frac{kJ}{kg} \right)\left(\frac{1kW}{1kJ/s} \right) = 1.55\,kW$$

Similarly, for the valve;

$$\dot{I}_{valve} = T_o \dot{m}(s_4 - s_3) = (294K)\left(6\frac{kg}{min} \right)\left(\frac{1min}{60s} \right)(0.33011 - 0.3113)\left(\frac{kJ}{kg} \right)\left(\frac{1kW}{1kJ/s} \right) = 0.5530\,kW$$

(d) The change in specific flow availability for refrigerant passing through the evaporator is:

$$e_{f1} - e_{f4} = (h_1 - h_4) - T_o(s_1 - s_4) = (243.40 - 82.90)\frac{kJ}{kg} - (294)K(0.9606 - 0.33011)\frac{kJ}{kgK} = -24.86\, kJ/kg$$

For the condenser:

$$e_{f3} - e_{f2} = (h_3 - h_2) - T_o(s_3 - s_2) = (82.90 - 295.13)\frac{kJ}{kgK} - (294)K(0.3113 - 1.0135)\frac{kJ}{kgK} = -5.78\frac{kJ}{kg}$$

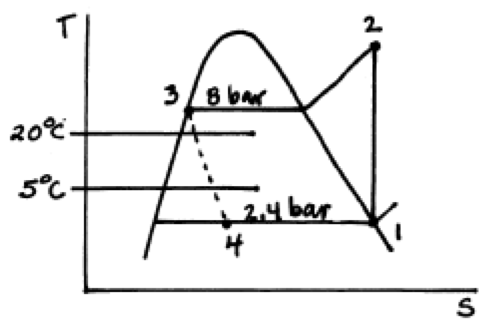
Comment: Although there is heat transfer to the refrigerant passing through the evaporator, the specific flow availability decreases. This can be explained by noting that the state of the working fluid moves closer to the dead state as it is heated at a temperature below T_o.

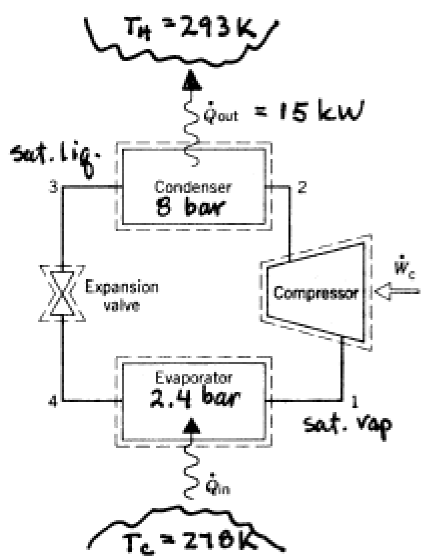
3) An ideal vapor-compression heat pump cycle with Refrigerant 134a as the working fluid provides 15 kW to maintain a building at 20^0C when the outside temperature is 5^0C. Saturated vapor at 2.4 bar leaves the evaporator, and saturated liquid at 8 bar leaves the condenser. Calculate

(a) The power input to the compressor, in kW
(b) The coefficient of performance.
(c) The coefficient of performance of a reversible heat pump cycle operating between thermal reservoirs at 20 and 5^0C

Solution:
Known: An ideal vapor-compression heat pump cycle uses Refrigerant 134a as the working fluid and provides a known energy output to heat a building. Data are known at various locations.

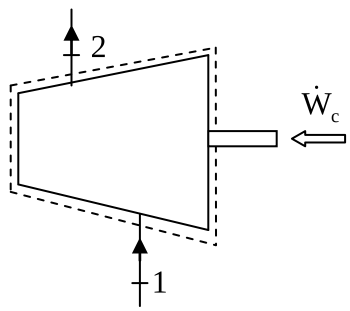
Schematic and Given Data:





Analysis: First, fix each of the principal states

Compressor :



Assumptions:

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- open system
- $\Delta KE = \Delta PE = 0$
- Neglect heat transfer (q=0)

$$\frac{dE_{c.v}}{dt} = \dot{Q}_{c.v.} - \dot{W}_{c.v.} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

$$0 = \dot{W}_c + \dot{m}_1 h_1 - \dot{m}_2 h_2$$

$$\sum \dot{m}_i = \sum \dot{m}_e$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\dot{W}_c = \dot{m}(h_2 - h_1)$$

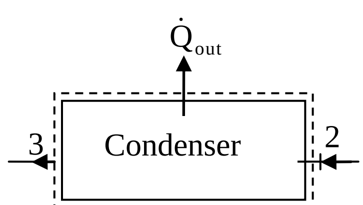
$$\frac{dS_{cv}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$$

$$0 = \dot{m}_1 s_1 - \dot{m}_2 s_2$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$s_1 = s_2$$

Condenser:



Assumptions:

- steady state steady flow process(SSSF)
- open system
- $\Delta KE = \Delta PE = 0$

- Neglect work (w = 0)

$$\frac{dE_{c.v}}{dt} = \dot{Q}_{c.v.} \underbrace{- \dot{W}_{c.v.}}_{0} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

$$0 = -\dot{Q}_{out} + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

$$\sum \dot{m}_i = \sum \dot{m}_e$$

$$\dot{m}_2 = \dot{m}_3 = \dot{m}$$

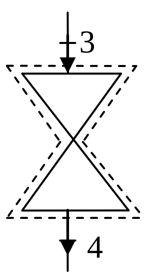
$$\dot{Q}_{in} = \dot{m}(h_2 - h_3)$$

State 1: $p_1 = 2.4$ bars, saturated vapor $\Rightarrow h_1 = 244.09$ kJ/kg, $s_1 = 0.9222$ kJ/kg.K

State 2: $p_2 = 8$ bars, $s_2 = s_1 \Rightarrow h_2 = 268.97$ kJ/kg

State 3: $p_3 = 8$ bars, saturated liquid $\Rightarrow h_3 = 93.42$ kJ/kg

Expansion valve:



Assumptions:

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- ΔKE = ΔPE =0
- Neglect heat transfer (q=0)
- Neglect work (w=0)

$$\frac{dE_{c.v}}{dt} = \underbrace{\dot{Q}_{c.v.}}_{0} - \underbrace{\dot{W}_{c.v.}}_{0} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

$$0 = \dot{m}_3 h_3 - \dot{m}_4 h_4$$

$$\sum \dot{m}_i = \sum \dot{m}_e$$

$$\dot{m}_3 = \dot{m}_4 = \dot{m}$$

$$h_3 = h_4$$

State 4: Throttling process $\Rightarrow h_3 = h_4 = 93.42 \text{ kJ/kg}$

(a) To determine the compressor, first find the mass flow rate from

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3)$$

or

$$\dot{m} = \frac{\dot{Q}_{out}}{h_2 - h_3} = \frac{15 \text{ kW}}{(268.97 - 93.42)\frac{\text{kJ}}{\text{kg}}}\left(\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right) = 0.08544 \text{ kg/s}$$

Thus,

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(0.08544\frac{\text{kg}}{\text{s}}\right)(268.97 - 244.09)\frac{\text{kJ}}{\text{kg}}\left(\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right)$$

$$= 2.126 \text{ kW}$$

(b) The coefficient of performance is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{15}{2.126} = 7.055$$

(c) For a reversible heat pump operating between reservoirs at

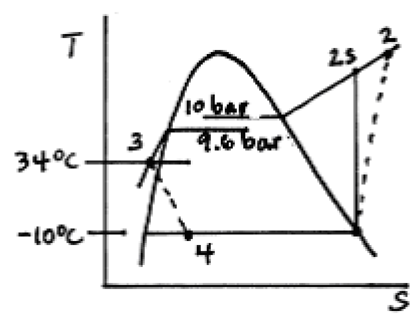
$T_H = 293 \text{ K}$ and $T_C = 278 \text{ K}$

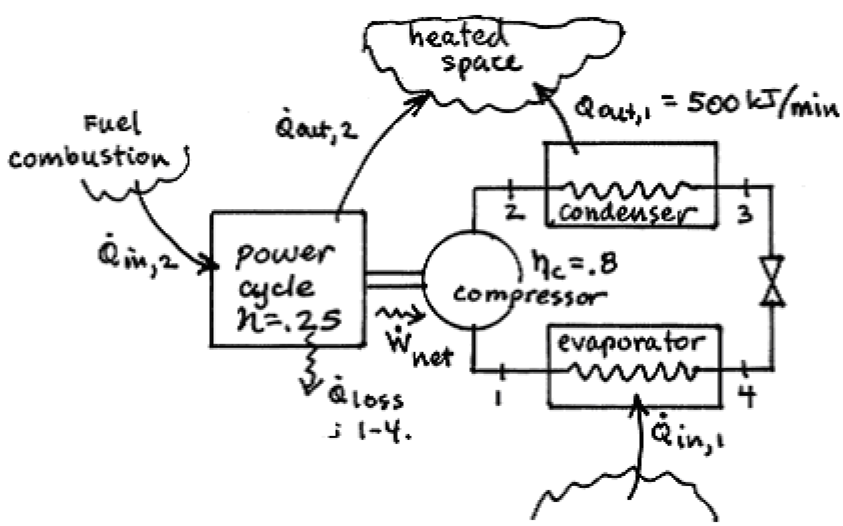
$$\gamma_{max} = \frac{T_H}{T_H - T_C} = \frac{293}{293 - 278} = 19.53$$

4) A vapor-compression heat pump with a heating capacity of 500 kJ/min is driven by a power cycle with a thermal efficiency of 25%. For the heat pump, Refrigerant 134a is compressed from saturated vapor at -10^{0}C to the condenser pressure of 10 bar. The isentropic compressor efficiency is 80%. Liquid enters the expansion valve at 9.6 bar, 34^{0}C. For the power cycle, 80% of the heat rejected is transferred to the heated space.

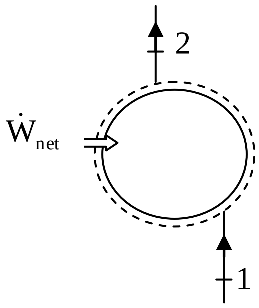
(a) Determine the power input to the heat pump compressor, in kW.
(b) Evaluate the ratio of the total rate that heat is delivered to the heated space to the rate of heat input to the power cycle. Discuss.

Solution:
Known: Refrigerant 134a is the working fluid in a vapor-compression heat pump driven by a power cycle. Operating data are specified for the heat pump and the power cycle.





Analysis: First, fix each state of the heat pump cycle.
Compressor:



Assumptions:

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- open system
- ΔKE = ΔPE =0
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$$\frac{dE_{c.v}}{dt} = \dot{Q}_{c.v.} - \dot{W}_{c.v.} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

$$0 = \left(\dot{W}_c\right)_s + \dot{m}_1 h_1 - \dot{m}_2 h_{2s}$$

$$\sum \dot{m}_i = \sum \dot{m}_e$$

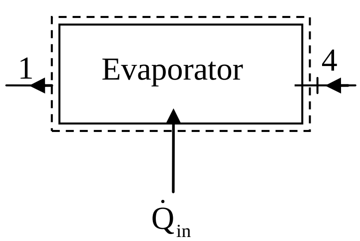
$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\left(\dot{W}_c\right)_s = \dot{m}(h_{2s} - h_1)$$

$$\left(\dot{W}_c\right)_a = \dot{m}(h_2 - h_1)$$

$$\eta = \frac{(\dot{W}_c)_s}{(\dot{W}_c)_a} = \frac{\left(h_{2s} - h_1\right)}{\left(h_2 - h_1\right)}$$

Evaporator:



Assumptions:

- steady state steady flow process(SSSF)
- open system
- ΔKE = ΔPE =0
- Neglect work (w = 0)

$$\frac{dE_{c.v}}{dt} = \dot{Q}_{c.v.} \underbrace{-\dot{W}_{c.v.}}_{0} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

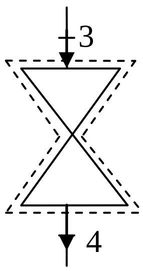
$$0 = \dot{Q}_{in} + \dot{m}_4 h_4 - \dot{m}_1 h_1$$

$$\sum \dot{m}_i = \sum \dot{m}_e$$

$$\dot{m}_1 = \dot{m}_4 = \dot{m}$$

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4)$$

Expansion valve:



Assumptions:

- steady state steady flow process(SSSF)
- open system
- $\Delta KE = \Delta PE = 0$
- Neglect heat transfer (q=0)
- Neglect work (w=0)

$$\frac{dE_{c.v}}{dt} = \underbrace{\dot{Q}_{c.v.}}_{0} - \underbrace{\dot{W}_{c.v.}}_{0} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

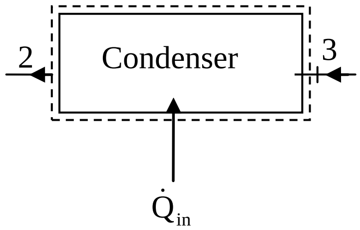
$$0 = \dot{m}_3 h_3 - \dot{m}_4 h_4$$

$$\sum \dot{m}_i = \sum \dot{m}_e$$

$$\dot{m}_3 = \dot{m}_4 = \dot{m}$$

$$h_3 = h_4$$

Condenser:



Assumptions:

- steady state steady flow process(SSSF)
- open system
- $\Delta KE = \Delta PE = 0$
- Neglect work (w = 0)

$$\frac{dE_{c.v}}{dt} = \dot{Q}_{c.v.} \underbrace{-\dot{W}_{c.v.}}_{0} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

$$0 = \dot{Q}_{in} + \dot{m}_3 h_3 - \dot{m}_2 h_2$$

$$\sum \dot{m}_i = \sum \dot{m}_e$$

$$\dot{m}_2 = \dot{m}_3 = \dot{m}$$

$$\dot{Q}_{in} = \dot{m}(h_2 - h_3)$$

State 1: $T_1 = -10^0 C$, saturated vapor $\Rightarrow h_1 = 241.34\ kJ/kg,\ s_1 = 0.9253\ kJ7kg.K$

State 2: For isentropic compression, $p_2 = 10$ bars, $s_{2s} = s_1 \Rightarrow h_{2s} = 274.63\ kJ/kg$

Using the compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 241.34\frac{kJ}{kg} + \frac{(274.63\,kJ/kg) - (241.24\,kJ/kg)}{0.8} = 282.95\ kJ/kg$$

State 3: $p_3 = 9.6$ bars, $T_3 = 34^0 C \Rightarrow$ compressed liquid; $h_3 \approx h_f(34^0 C) = 97.31\ kJ/kg$

State 4: Throttling process $\Rightarrow h_4 = h_3 = 97.31\ kJ/kg$

(a) The mass flow rate of refrigerant is

$$\dot{m} = \frac{\dot{Q}_{out,1}}{h_2 - h_3} = \frac{500\ kJ/min}{(282.95 - 97.31)\ kJ/kg}\left(\frac{1\ min}{60\ s}\right) = 0.04489\ kg/s$$

The compressor power becomes

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(0.04489\ \frac{kg}{s}\right)(282.95 - 241.34)\frac{kJ}{kg}\left(\frac{1\ kW}{1\ kJ/s}\right)$$

$$= 1.868\ kW$$

(b) For the power cycle, $\eta_c = 0.25$. With $\dot{W}_{power\ cycle} = \dot{W}_{heat\ pump} = 1.868\ kW$

$$\dot{Q}_{in,2} = \frac{\dot{W}_{cycle}}{\eta} = 7.472$$

The total rejected is,

$$\dot{Q}_{rej} = 7.472\ kW - 1.868\ kW = 5.604\ kW$$

Thus,

$$\dot{Q}_{out,2} = (0.8)\dot{Q}_{rej} = 4.483\ kW$$

Finally, $\frac{\dot{Q}_{out,1} + \dot{Q}_{out,2}}{\dot{Q}_{in,2}} = \frac{\left(500\frac{kJ}{min}\frac{1 min}{60 s}\frac{1 kW}{1 kJ}\right) + 4.483 kW}{7.472\ kW} = 1.715$

COMMENT: The engine-driven heat pump delivers more energy to the heated space than could be obtained by burning fuel directly.

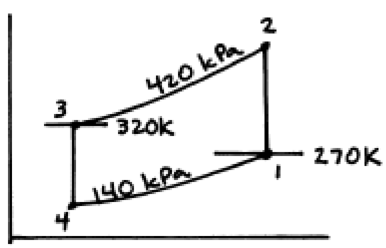
5) Air enters the compressor of an ideal Brayton refrigeration cycle at 140 kPa, 270 K, with a volumetric flow rate of 1 m^3/s, and is compressed to 420 kPa. The temperature at the turbine inlet is 320 K. Determine

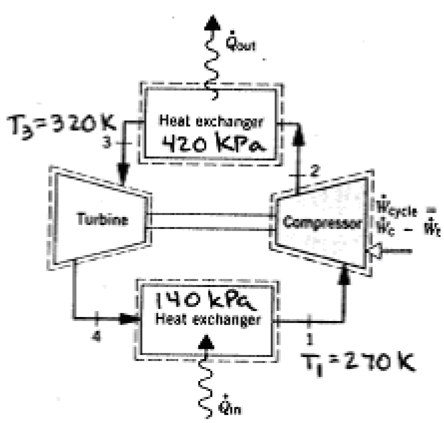
(a) The net power input, in kW
(b) The refrigerating capacity, in kW.
(c) The coefficient of performance.
(d) The coefficient of performance of a reversible refrigeration cycle operating between reservoirs at T_C = 270 K and T_H = 320 K.

Solution:

Known: Air is the refrigerant in an ideal Brayton refrigeration cycle. Data are known at various locations and the volumetric flow rate at the compressor inlet is given.

Schematic and Given Data:





Analysis: First, fix each of the principal states

State 1: $T_1 = 270\ \text{K} \Rightarrow h_1 = 270.11\ \text{kJ/kg},\ p_{r1} = 0.9590$

State 2: $p_{r2} = p_{r1}\left(\frac{p_2}{p_1}\right) = 0.9590 \times \left(\frac{420\,\text{kPa}}{140\,\text{kPa}}\right) = 2.877 \Rightarrow h_2 = 370.10\ \text{kJ/kg}$

State 3: $T_3 = 320\ \text{K} \Rightarrow h_3 = 320.29\ \text{kJ/kg},\ p_{r3} = 1.7375$

State 4: $p_{r4} = p_{r3}\left(\frac{p_4}{p_3}\right) = 1.7375\left(\frac{140\,\text{kPa}}{420\,\text{kPa}}\right) = 0.5792 \Rightarrow h_4 = 233.61\ \text{kJ/kg}$

(a) The mass flow rate is

$$\dot{m} = \frac{(A\forall)_1}{v_1} = \frac{(A\forall)_1 p_1}{RT_1} = \frac{(1\ \text{m}^3/\text{s})(140\,\text{kPa})}{\left(\frac{8.314\ \text{kJ}}{28.97\ \text{kg.K}}\right)(270\,\text{K})}\left(\frac{10^3\ \text{N/m}^2}{1\ \text{kPa}}\right)\left(\frac{1\ \text{kJ}}{10^3\ \text{N.m}}\right) = 1.807\ \text{kg/s}$$

Thus, the net power is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p$$
$$\dot{W}_{cycle} = \dot{m}\left[(h_2 - h_1) - (h_3 - h_4)\right]$$
$$= \left(1.807\ \frac{\text{kg}}{\text{s}}\right)\left[(370.1 - 270.11) - (320.29 - 233.61)\right]\frac{\text{kJ}}{\text{kg}}\left(\frac{1\ \text{kW}}{1\text{kJ/s}}\right)$$
$$= 24.05\ \text{kW}$$

(b) The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(1.807\frac{\text{kg}}{\text{s}}\right)(270.11 - 233.61)\frac{\text{kJ}}{\text{kg}} = 65.96\ \text{kW}$$

(c) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}} = \frac{65.96}{24.05} = 2.743$$

(d) For a reversible cycle operating between thermal reservoirs at 270 K and $T_H = 320\ \text{K}$ is

$$\beta_{max} = \frac{T_C}{T_H - T_C} = \frac{270}{320 - 270} = 5.4$$