

CANKAYA UNIVERSITY
FACULTY OF ENGINEERING AND ARCHITECTURE
MECHANICAL ENGINEERING DEPARTMENT
ME 212 THERMODYNAMICS II

CHAPTER 11

EXAMPLES SOLUTION

1) The pressure within a 23.3 m³ tank should not exceed 105 bar. Check the pressure within the tank if filled with 1000 kg of water vapor maintained at 360^oC using the

- a) ideal gas equation
- b) van der Waals equation
- c) Redlich-Kwong equation
- d) Compressibility chart
- e) Steam tables

Solution:

Analysis:

a) Ideal gas equation:

$$p = \frac{RT}{v} = \frac{(8314/18.02) \left(\frac{\text{N.m}}{\text{kg.K}} \right) (663\text{K})}{\left(\frac{23.3}{1000} \right) \left(\frac{\text{m}^3}{\text{kg}} \right)} \left(\frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right) = 125.29 \text{ bars (25\% high)}$$

b) Van der Waals equation with a and b from Table A-24:

$$p = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}^2} = \left[\frac{\left(8314 \frac{\text{N.m}}{\text{kmol.K}} \right) (633 \text{ K})}{(0.42 - 0.0305) \frac{\text{m}^3}{\text{kmol}}} \right] \left(\frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right) - \frac{5.531 \text{ bar}(\text{m}^3 / \text{kmol.K})^2}{\left(0.42 \frac{\text{m}^3}{\text{kmol}} \right)^2}$$

$$= 103.76 \text{ bars (24\% high)}$$

c) Redlich-Kwong equation with a and b from Table A-24:

$$p = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}(\bar{v} + b)T^{1/2}} = \frac{\left(8314 \frac{\text{N.m}}{\text{kmol.K}}\right)(633\text{K})}{(0.42 - 0.02111) \frac{\text{m}^3}{\text{kmol}}} \left(\frac{1 \text{ bar}}{10^5 \text{ N/m}^2}\right) - \frac{142.59 \text{ bar} \left(\frac{\text{m}^3}{\text{kmol}}\right)^2 \text{K}^{1/2}}{\left(0.42 \frac{\text{m}^3}{\text{kmol}}\right)(0.42 + 0.02111) \left(\frac{\text{m}^3}{\text{kmol}}\right)(633 \text{ K})^{1/2}} = 101.34 \text{ bars (1% high)}$$

d) Compressibility chart:

From Table A-1, $T_c = 647.3 \text{ K}$, $p_c = 220.9 \text{ bars}$, thus; $T_R = \frac{633}{647.3} = 0.978$

And;

$$v'_R = \frac{\bar{v}p_c}{\bar{R}T_c} = \frac{(0.42 \text{ m}^3 / \text{kmol})(220.9 \times 10^5 \text{ N/m}^2)}{\left(8314 \frac{\text{N.m}}{\text{kmol.K}}\right)(647.3 \text{ K})} = 1.724$$

Then, Fig A-1 gives $Z \cong 0.81$ so;

$$p = \frac{ZRT}{p} = 0.81(125.29) = 101.48 \text{ (1.5% high)}$$

e) Steam tables gives 100 bars.

2) Using $\left(\frac{\partial T}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T$ check the consistency of

(a) the steam tables at 2 MPa, 400 °C

(b) the refrigerant 134a at 2 bar, 50 °C

3) Evaluate the partial derivative $\left(\frac{\partial s}{\partial v}\right)_T$ for water vapor at a fixed state at temperature of 240°C and specific volume of 0.4646 m³/kg.

- a) Use Redlich-Kwong equation of state and an appropriate Maxwell relation.
 b) Check the value obtained using steam table data

Solution:

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v \quad \text{and} \quad p = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}(\bar{v} + b)\sqrt{T}}$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{\bar{R}}{\bar{v} - b} + \frac{a}{2\bar{v}(\bar{v} + b)T^{3/2}}$$

At T = 240°C = 513 K

$$\bar{v} = \left(0.4646 \frac{\text{m}^3}{\text{kg}}\right) \left(\frac{18.02 \text{ kg}}{1 \text{ kmol}}\right) = 8.372 \frac{\text{m}^3}{\text{kmol}}$$

Use Table A-24:

$$a = 142.59 \left(\frac{\text{m}^3}{\text{kmol}}\right)^2 \sqrt{\text{K}}$$

$$b = 0.0211 \text{ m}^3 / \text{kmol}$$

Substitute into equation:

$$\left(\frac{\partial p}{\partial T}\right)_v = 1.0043 \frac{\text{kJ}}{\text{m}^3 \cdot \text{K}}$$

b)

T = 240°C		
P (bar)	s (kJ/kg.K)	v (m ³ /kg)
1	7.9949	2.359
1.5	7.9052	1.57
3	7.4774	0.781
5	7.2307	0.4646
7	7.0641	0.3292
10	6.8817	0.2275

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data =  $\begin{pmatrix} 2.359 & 7.9949 \\ 1.57 & 7.8052 \\ 0.781 & 7.4774 \\ 0.4646 & 7.2307 \\ 0.3292 & 7.0641 \\ 0.2275 & 6.8817 \end{pmatrix}$ 

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Out[54]= {{2.359, 7.9949}, {1.57, 7.8052}, {0.781, 7.4774},
          {0.4646, 7.2307}, {0.3292, 7.0641}, {0.2275, 6.8817}}

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In[55]=

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p1 = ListPlot[data, PlotRange -> {{0, 3}, {6, 8}}, PlotStyle -> PointSize[0.02]];

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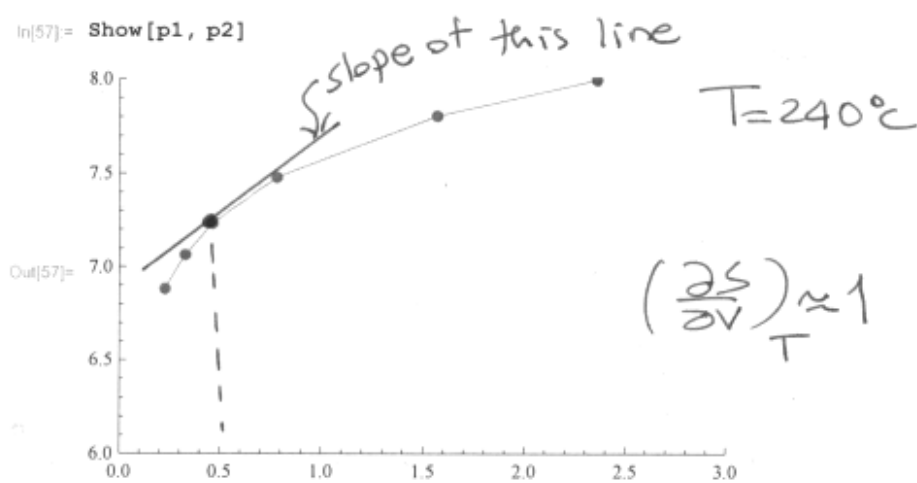
In[56]= p2 = ListPlot[data, PlotRange -> {{0, 3}, {6, 8}}, Joined -> True];

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In[57]= Show[p1, p2]

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4) Develop expressions for the volume expansivity β and the isothermal compressibility κ for

- an ideal gas
- a gas whose equation of state is $p(v-b) = RT$
- a gas obeying the van der Waals equation

Solution:

Analysis:

$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p \quad \text{and} \quad \kappa = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_T$$

a) Ideal gas:

$$v = RT/p, \quad \left(\frac{\partial v}{\partial T} \right)_p = \frac{R}{p} \quad \text{and} \quad \left(\frac{\partial v}{\partial p} \right)_T = \frac{-RT}{p^2} \quad \text{Thus;}$$

$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p = \frac{1}{v} \left(\frac{R}{p} \right) = \frac{1}{T}$$

$$\kappa = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_T = -\frac{1}{v} \left(\frac{-RT}{p^2} \right) = \frac{1}{p} \left(\frac{RT}{pv} \right) = \frac{1}{p}$$

b) $v = (RT/p) + b$

$$\left(\frac{\partial v}{\partial T} \right)_p = \frac{R}{p} \quad \text{and} \quad \left(\frac{\partial v}{\partial p} \right)_T = \frac{-RT}{p^2} \quad \text{. Thus:}$$

$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p = \frac{1}{v} \left(\frac{R}{p} \right) = \frac{R}{v} \left(\frac{v-b}{RT} \right) = \frac{1}{T} \left(\frac{v-b}{v} \right)$$

$$\kappa = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_T = -\frac{1}{v} \left(\frac{-RT}{p^2} \right) = \frac{1}{p} \left(\frac{RT/v}{RT/(v-b)} \right) = \frac{1}{p} \left(\frac{v-b}{v} \right)$$

Note: when $b = 0$ these expressions reduce to those of part (a).

c) From Van der Waals equation:

$$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

As the Van der Waals equation is not explicit in v , the required partial derivatives are not so easily found as in parts (a) and (b). Thus an expression for $(\partial v / \partial T)_p$ is obtained as:

$$\left(\frac{\partial v}{\partial T} \right)_p = \frac{-R/(v-b)}{[2a/v^3 - RT/(v-b)^2]} \quad \text{so} \quad \beta = \frac{-R(v-b)v^2}{2a(v-b)^2 - RTv^3}$$

$$\left(\frac{\partial v}{\partial p} \right)_T = \frac{-(\partial v / \partial T)_p}{(\partial p / \partial T)_v} = \frac{\frac{R/(v-b)}{[2a/v^3 - RT/(v-b)^2]}}{R/(v-b)} = \frac{1}{[2a/v^3 - RT/(v-b)^2]}$$

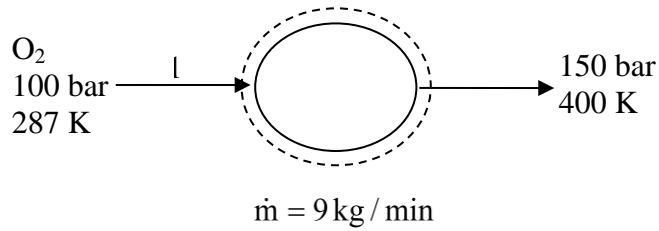
Thus;

$$\kappa = \frac{1}{v} \frac{\partial v}{\partial p}_T = \frac{1}{v[2a/v^3 - RT/(v-b)^2]} = \frac{v^2(v-b)^2}{[2a(v-b)^2 - RTv^3]}$$

5) Oxygen (O_2) enters a control volume operating at steady state with a mass flow rate of 9 kg/min at 100 bar, 287 K and is compressed adiabatically to 150 bar, 400 K. Determine the power required, in kW, and the rate of entropy production, in kW/K. Ignore kinetic and potential energy effects.

Solution:

Schematic and Given Data:



Analysis:

Reducing mass, energy and entropy balances using the given assumptions,

$$\frac{dE}{dt} = \dot{Q} - \dot{W} + \sum \dot{m}_i h_i - \sum \dot{m}_e h_e$$

$$\Rightarrow \frac{\dot{W}_t}{\dot{m}} = h_1 - h_2$$

and

$$\frac{dS_{cv}}{dt} = \sum \underbrace{\frac{\dot{Q}_j}{T_j}}_{=0} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{\sigma}_{cv}$$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1$$

From tables it is obtaining for O₂ that;

$T_c = 154 \text{ K}$, $p_c = 50.5 \text{ bar}$. Thus:

$$p_{R1} = \frac{p_1}{p_c} = \frac{100}{50.5} = 1.98, \quad T_{R1} = \frac{T_1}{T_c} = \frac{287}{154} = 1.86$$

$$p_{R2} = \frac{p_2}{p_c} = \frac{150}{50.5} = 2.97, \quad T_{R2} = \frac{T_2}{T_c} = \frac{400}{154} = 2.60$$

Then, tabulated enthalpy and entropy departure data gives:

$$\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right)_1 = 0.55, \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right)_2 = 0.40, \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}} \right)_1 = 0.23, \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}} \right)_2 = 0.08$$

For O₂, tables gives: $\bar{h}_1 - \bar{h}_2 = 8355 - 11711 = -3356 \text{ kJ/kmol}$.

Then:

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{\bar{h}_1^* - \bar{h}_2^* - \bar{R}T_c \left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right)_1 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right)_2 \right]}{M} = \frac{-3356 - [(8.314)(154)][0.55 - 0.40]}{32}$$

$$\Rightarrow \frac{\dot{W}_{cv}}{\dot{m}} = -110.9 \text{ kJ/kg}$$

Thus,

$$\dot{W}_{cv} = \left(9 \frac{\text{kg}}{\text{min}} \right) \left(-110.9 \frac{\text{kJ}}{\text{kg}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \left(\frac{1 \text{ kW}}{1 \text{ kJ/s}} \right) = -16.64 \text{ kW}$$

With; $\Delta \bar{s}^* = \bar{s}^o(T_2) - \bar{s}^o(T_1) - \bar{R} \ln \frac{P_2}{P_1}$ and data from tables:

$$\Delta \bar{s}^* = 213.765 - 203.91 - \left[8.314 \ln \left(\frac{150}{100} \right) \right] = 6.484 \text{ kJ/kmol.K}$$

Accordingly;

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = \frac{\Delta \bar{s}^* - \bar{R} \left[\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}} \right)_2 - \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}} \right)_1 \right]}{M} = \frac{6.484 - 8.314(0.08 - 0.23)}{32} = 0.2416 \frac{\text{kJ}}{\text{kg.K}}$$

Finally,

$$\Rightarrow \dot{\sigma}_{cv} = \left(9 \frac{\text{kg}}{\text{min}} \right) \left(0.2416 \frac{\text{kJ}}{\text{kg.K}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \left(\frac{1 \text{ kW}}{1 \text{ kJ/s}} \right) = 0.036 \frac{\text{kW}}{\text{K}}$$

6) A closed, rigid, insulated vessel having a volume of 0.142 m^3 contains oxygen (O_2) initially at 100 bar, 7°C . The oxygen is stirred by a paddle wheel until the pressure becomes 150 bar. Determine the

- (a) final temperature, in $^\circ\text{C}$.
- (b) work, in kJ.
- (c) amount of exergy destroyed in the process, in kJ.

Let $T_0 = 7^\circ\text{C}$

7) One kmol of gaseous mixture occupies a volume of 0.111 m^3 at 100°C . The mixture consists of 69.5% carbon dioxide and 30.5% ethylene (C_2H_4) (molar-basis). Estimate the mixture pressure using

- the ideal gas equation of state
- Kay's rule together with the generalized compressibility chart.
- The additive pressure rule together with the generalized compressibility chart.
- The van der Waals equation together with mixture values for the constants a and b

Solution:

Analysis:

CO_2 C_2H_4	$V = 0.111 \text{ m}^3$ $T = 373 \text{ K}$ $\text{CO}_2 : y_1 = 0.695,$ $\text{C}_2\text{H}_4 : y_2 = 0.305$
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- a) Ideal gas equation of state:

$$p = \frac{\bar{n}RT}{V} = \frac{(1 \text{ kmol})(8314 \text{ N.m/kmol.K})(373 \text{ K})}{0.111 \text{ m}^3} \left(\frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right) = 279.4 \text{ bar}$$

- b) Kays Rule with Z chart:

With data from properties tables;

$$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.695)(304 \text{ K}) + (0.305)(283 \text{ K}) = 297.6 \text{ K}$$

$$p_c = y_1 p_{c1} + y_2 p_{c2} = (0.695)(73.9 \text{ bar}) + (0.305)(51.2 \text{ bar}) = 66.98 \text{ bar}$$

Thus;

$$T_R = \frac{T}{T_c} = \frac{373 \text{ K}}{297.6 \text{ K}} = 1.25, \quad v'_R = \frac{\bar{v} p_c}{RT_c} = \frac{(0.111 \text{ m}^3 / \text{kmol})(66.98 \times 10^5 \text{ Pa})}{\left(8314 \frac{\text{N.m}}{\text{kmol.K}} \right) (297.6 \text{ K})} = 0.3,$$

Accordingly, $Z \approx 0.61$, so

$$p = Z \frac{\bar{n}RT}{V} = Z p_{\text{ideal}} = (0.61)(279.4 \text{ bar}) = 170.4 \text{ bar}$$

- c) Additive pressure rule:

$$Z = y_1 Z_1(T, V) + y_2 Z_2(T, V)$$

where;

$$T_{R1} = \frac{373}{304} = 1.227, \quad v'_{R1} = \frac{\left(\frac{0.111}{0.695}\right)(73.9 \times 10^5)}{(8314)(304)} = 0.467 \Rightarrow Z_1 = 0.68$$

$$T_{R2} = \frac{373}{283} = 1.318, \quad v'_{R2} = \frac{\left(\frac{0.111}{0.305}\right)(51.2 \times 10^5)}{(8314)(283)} = 0.792 \Rightarrow Z_2 = 0.81$$

Accordingly;

$$Z = y_1 Z_1(T, V) + y_2 Z_2(T, V) = (0.695)(0.68) + (0.305)(0.81) = 0.72$$

Then,

$$p = Z \frac{n\bar{R}T}{V} = Z p_{\text{ideal}} = (0.72)(279.4) = 201.2 \text{ bar}$$

d) Van der Waals equation:

Van der Waals constants for CO₂ can be obtained from tables as:

$$a_1 = 3.647 \text{ bar} \left(\frac{\text{m}^3}{\text{kmol}} \right)^2 \quad b_1 = 0.0428 \text{ m}^3 / \text{kmol}$$

The respective values for C₂H₄ can be obtained from critical data:

$$a_2 = \frac{27}{64} \frac{\bar{R}^2 T_c^2}{p_c} = \frac{27}{64} \frac{(8314)^2 (283)^2}{(51.2 \times 10^5)} \frac{1}{10^5} = 4.561 \text{ bar} \left(\frac{\text{m}^3}{\text{kmol}} \right)^2$$

$$b_2 = \frac{\bar{R}T_c}{8p_c} = \frac{(8314)(283)}{(8)(51.2 \times 10^5)} = 0.0574 \frac{\text{m}^3}{\text{kmol}}$$

Then,

$$a = \left[\sum_{i=1}^j y_i \sqrt{a_i} \right]^2 = \left[y_1 a_1^{1/2} + y_2 a_2^{1/2} \right]^2$$

$$= \left[(0.695)(3.647)^{1/2} + (0.305)(4.561)^{1/2} \right]^2 = 3.9147 \text{ bar} \left(\frac{\text{m}^3}{\text{kmol}} \right)^2$$

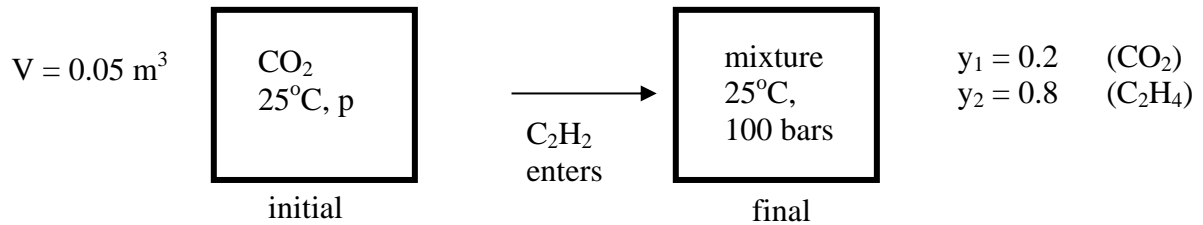
$$b = \sum_{i=1}^j y_i b_i = y_1 b_1 + y_2 b_2 = (0.695)(0.0428) + (0.305)(0.0574) = 0.04725 \text{ m}^3 / \text{kmol}$$

With these values;

$$p = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}^2} = \frac{(8314)(373)}{(0.111 - 0.04725)} - \frac{3.9147}{(0.111)^2} = 168.7 \text{ bar}$$

8) A rigid vessel having a volume of 0.05 m^3 initially contains carbon dioxide gas at temperature 25°C and pressure p . Ethylene gas is allowed to flow into the tank until a mixture consisting of 20% carbon dioxide and 80% ethylene (molar basis) exits within the tank at a temperature of 25°C and a pressure of 100 bar. Determine the pressure p , in bar, using Kay's rule together with the generalized compressibility chart.

Schematic and Given Data:



Analysis:

The initial pressure p can be found using $p = p_R p_c$ where p_R is found from the compressibility chart using $T_R = T/T_c$, where T_c is the critical temperature of CO_2 and,

$v'_R = \left(\frac{V}{n}\right) \frac{p_c}{RT_c}$ where p_c is the critical pressure of CO_2 and n is the number of kmol of CO_2 present initially. To find n , consider the final condition consisting of the mixture.

$$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.2)(304 \text{ K}) + (0.8)(283 \text{ K}) = 287.2 \text{ K}$$

$$p_c = y_1 p_{c1} + y_2 p_{c2} = (0.2)(73.9 \text{ bar}) + (0.8)(51.2 \text{ bar}) = 55.74 \text{ bar}$$

$$\text{Then, } T_R = \frac{T}{T_c} = \frac{298}{287.2} = 1.038, \quad p_R = \frac{p}{p_c} = \frac{100}{55.74} = 1.794 \Rightarrow Z \cong 0.32$$

$$\text{Then, the amount of mixture is obtained using } Z = \frac{pV}{n_{\text{mix}} RT} \text{ or } n_{\text{mix}} = \frac{p_{\text{final}} V}{ZRT_{\text{final}}}$$

Since the amount of CO_2 present in the final mixture is $(0.2 n_{\text{mix}})$, the amount of CO_2 present initially is $n = 0.2 n_{\text{mix}}$ or $n = 0.2 \left(\frac{p_{\text{final}} V}{ZRT_{\text{final}}} \right)$

Thus for the initial condition, $T_R = 298/304 = 0.98$ and

$$v'_R = \frac{(V/n)p_c}{\bar{R}T_c} = \frac{Vp_c}{\left[0.2\left(\frac{p_{\text{final}} V}{Z\bar{R}T_{\text{final}}}\right)\right]\bar{R}T_c} = \left(\frac{Z}{0.2}\right)\left(\frac{p_c}{p_{\text{final}}}\right)\left(\frac{T_f}{T_c}\right) = \left(\frac{0.32}{0.2}\right)\left(\frac{73.9}{100}\right)\left(\frac{298}{304}\right) = 1.16$$

Then from compressibility chart gives $p_R \approx 0.63$, giving
 $p = p_R p_c = (0.63)(73.9) = 46.6 \text{ bar}$